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1958

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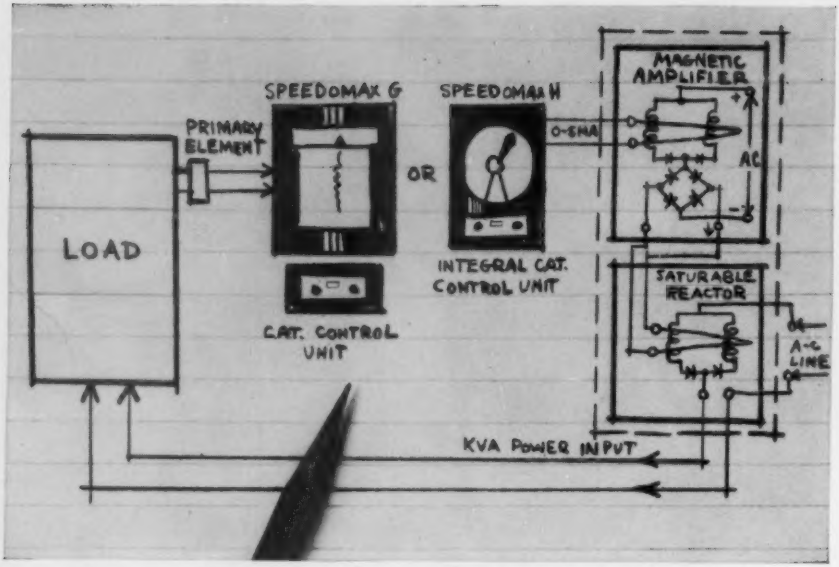

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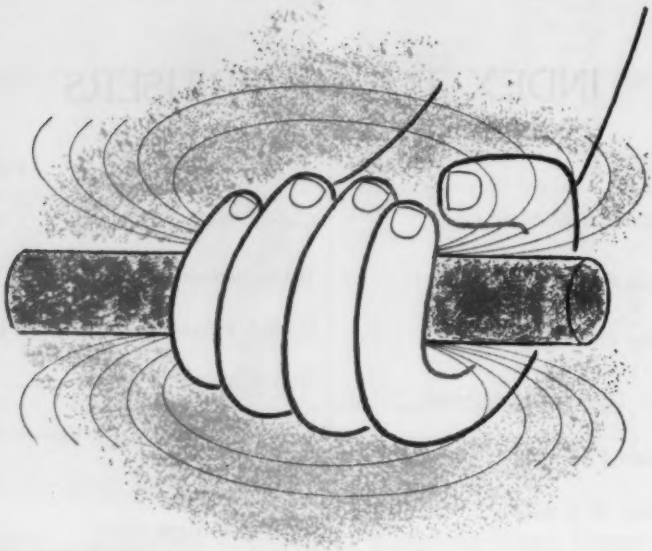

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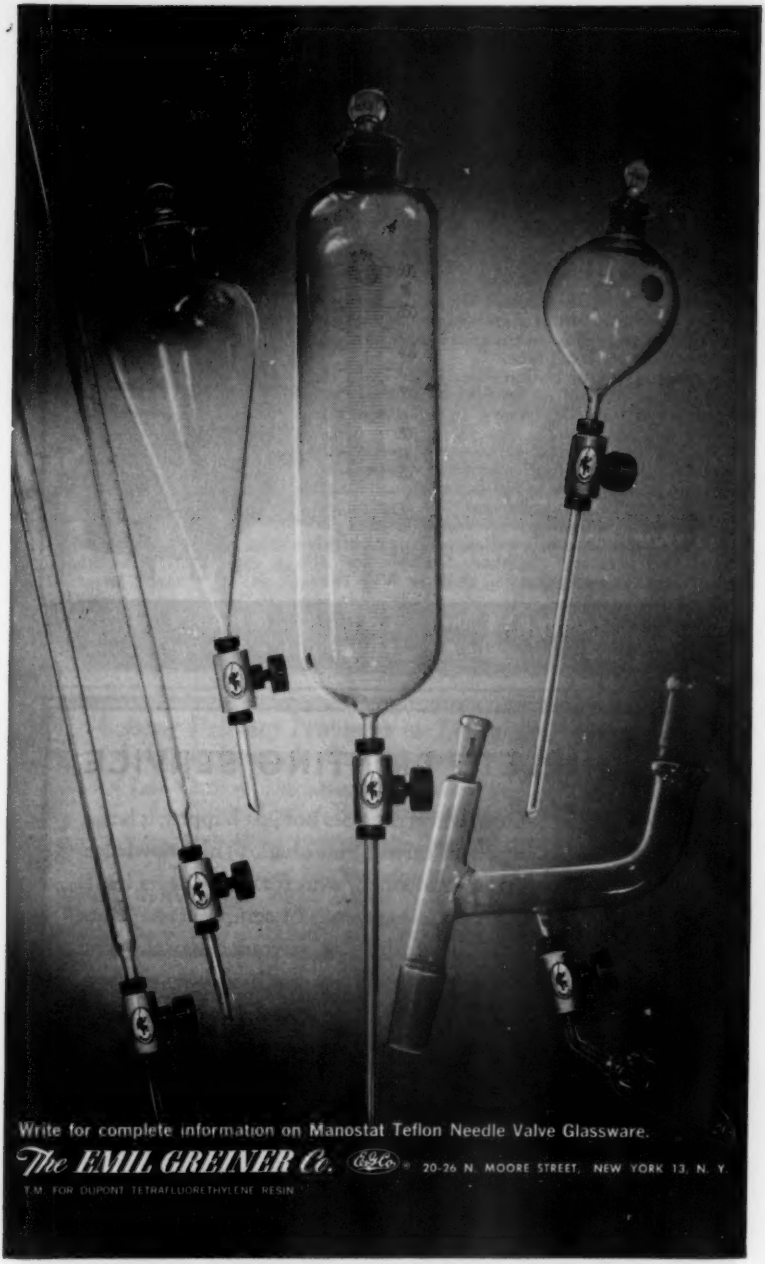
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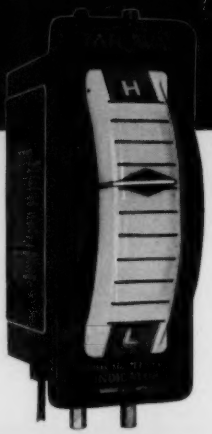

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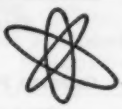

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Vol. 265 APRIL, 1958 No. 4

SAFE ATOMS *

BY

C. ROGERS McCULLOUGH [1]

INTRODUCTION

My objective tonight is to talk to you about atomic energy, its advantages, future promise, and also the dangers that go along with it. The title of my talk, "Safe Atoms," is really true. Atoms, that is radioactive atoms, are indeed safe when handled under the restrictions and with the precautions that we are currently taking. But like many a title, this title can be misleading. We must ask ourselves just what do we mean by "safe." In the course of our discussion tonight, I want to talk about natural radioactivity, the radiation that has been with us since the world began and under which the human race has developed. I want to talk about the fallout which has occurred from weapons testing, I want to talk about the radioactive material which is released when we work with atomic reactors and radioisotopes. I hope to touch upon the consequences of nuclear war very briefly. But most of all I want to tell you of the precautions that we take to protect employees and all of the people against damage by radioactivity when we use these radioisotopes and when we use reactors. I hope to show you how some of these protective measures are applied and why they work. Most of all I hope to put in perspective the possible danger of radioactivity versus the advantages atomic energy can bring us without incurring unnecessary hazards.

WHAT IS SAFETY?

In this world, there are few if any absolutes and, therefore, there is no such thing as perfect safety. If you want to be absolutely sure that

* The 1957 Philip C. Staples Lecture, delivered at the Stated Meeting of The Franklin Institute, Wednesday, November 20, 1957, in the Lecture Hall.

[1] Member, A. E. C. Advisory Committee on Reactor Safeguards, Washington, D. C.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

no one will be hurt by radioactivity, then we had better stop any work in the field of atomic energy right now. This applies not only to nuclear weapons and their testing but also to the use of reactors and radioisotopes in medicine. Do any of you want to give up the preferred treatment for hyperthyroidism with radioactive iodine 131? Do you wish to give up the preferred treatment for polycythemia vera using radioactive phosphorus 32? Let me remind you that this is the only treatment that I know of that has any proven effects. Do you wish to give up the extremely accurate and convenient method of diagnosing thyroid troubles using radioactive iodines? Do you wish to give up one of the most promising controls of cancer of the peritoneal cavity using radioactive gold 198? Or turning to industrial uses, would you be willing to forego the use of radioactive gauging methods for sheet steel, rubber, etc., using strontium 90? Do you want to give up the highly effective and convenient radiographing of structures and vessels made possible by radioactive Cobalt 60 and Iridium 192? In answering these questions we should remember that because we are able to test materials nondestructively we save money in construction, and we guard against failures which might result in injury or death to employees or the public. By the same token these radioactive medical treatments, although far from perfect, are effective in controlling the diseases mentioned. It should be remembered that the industrial and medical use of radioisotopes is in its infancy and we have every right to expect dramatic and important new uses in the future.

In using radiation we do incur some risks but I hope to show you how very low these risks are and how safe atomic energy is as compared with other fields. The major atomic energy activity has been going on for about thirteen years and something more than 300,000 people have been employed in it (1).[2] In all this time and with all these people there have been only two fatalities due to radiation and these occurred, one in 1945 and the other in 1946. There have been a few over-exposures which might be classed as injuries and there have been perhaps a few more exposures to radiation over the permissible level for employees or the public. But it must be emphasized that the vast majority of the workers and the public have not been exposed to radiation at a level greater than a small fraction of the levels which are considered to be safe and permissible. Another way of measuring the safety of atomic energy is to realize that its rate of accidents from *all causes* is next to the lowest and only one-third of the average of all industry (2). And please remember that these accidents are the usual ones like falling down steps, dropping objects on the foot, hitting the thumb with a hammer, etc., and practically none are due to radiation. Another way of putting it is to say that during all of the years that we have operated nuclear reactors in this country not a single person, either employee or public,

[2] The boldface numbers in parentheses refer to the references appended to this paper.

has been injured by these machines in regular operation. This safety record is so good that it bothers us. In spite of all of the precautions which we take, and I hope to convince you that we are very careful, it just does not seem possible to keep on with such a near perfect record. I am sorry to say it but I think we must be prepared to expect some accidents and some injuries. I wish it were not so.

At times we have been accused of being overly cautious and as a result holding back atomic energy for peaceful purposes by too great expense. We have and continue to consider this. Honestly I do not believe this is so, except possibly in a few isolated cases. It must be remembered that to shield the enormous amounts of radioactivity down to even the lethal level requires thick layers of steel and concrete. To bring this down to absolutely safe levels only requires a little more expense. Moreover we firmly believe we must not be reckless with possible injury to living generations or those to come.

THE NATURE OF RADIATION

At this point I would like to talk briefly about the nature of radiation and the way we protect people from it. I have here a geiger counter set up which is recording the natural radiation, or background as it is called, to which we are all subjected and which has been present ever since the world began. I am sorry but we have to get used to the units which are used in this business and radiation background is measured in millirem, that is 1/1000 of a rem, and a rem is a unit of radiation effect on human beings. The average natural background from all sources is about 160 millirem per year. Now we must add to this the amount of radiation which we would expect to get from medical diagnosis or treatment. Obviously this varies from person to person and from time to time, but a good average is 160 millirem per year. Let me point out, however, that certain X-ray procedures give as much as 6000 to 9000 milliroentgens for each picture and the 9000 value is for X-ray for certain conditions in pregnancy (3). This occasional click or flash of the light represents the average background which, as I said, was 160 milliroentgens a year. For our purposes there are four kinds of radiation—neutrons, gamma rays, beta rays, and alpha particles. Neutrons are a bit difficult to demonstrate and, therefore, I did not bring a source of neutrons with me.

However, here I have a gamma source and you will notice as I bring the probe of the instrument closer to it that the instrument indicates either on the dial or by the noise or the rapidity of the flashes the presence of these gamma rays. Notice also that one protection against the gamma radiation is distance. When my probe is far away there is practically no effect. As it gets closer and closer the probe gets greater and greater amounts of radiation. For the mathematically minded, this goes as the inverse square of the distance. At large distances the absorption by the air becomes appreciable. Let me illustrate shielding.

If I interpose these lead strips between the gamma source and the probe, practically all of the radiation is cut out. Notice that putting a piece of aluminum has practically no effect. This gamma source is very low level. I could carry it in my pocket without worrying but please notice that those of us trained in the atomic energy field simply would not think of carrying this around in our pockets even though we know it is such a weak source that it would do no harm. Our policy is to avoid radiation exposures of any kind as far as we can.

Next I have here a beta source. Again this is such a low level that it could be carried in my pocket without harm but again we simply do not do this sort of thing. Now notice that as I bring the probe close to this source with a cap on it, which had no effect on the gamma source, we get practically no effect. If I take the cap off, the counter shows a rather pronounced effect. Notice again how distance is effective in reducing the amount of radiation. The radiation in this case falls off approximately as the inverse square of the distance, but here the absorption of the air is appreciable. Now let me demonstrate shielding. If I interpose one layer of cardboard, the level is cut down, two sheets still more, three, etc. Now note, instead of the cardboard let me insert a sheet of aluminum, which had no appreciable effect on the gamma rays. In this case it cuts out substantially all of the beta rays.

Now finally, I have a source of alpha rays. Notice the radiation falls off very rapidly with distance. Here air absorption is very important. Please notice that there is a very thin film of plastic. By covering the alpha source with this, the radiation is completely absorbed.

Now, I would like to show you some common sources of radiation. I have here a luminous dial wrist watch. Please notice that this is just about as effective as a radiation source as my demonstration sources. Notice that the radiation level on the back of the watch is about half of that through the glass. Actually, the radiation given off by this dial is a mixture of betas and gammas. Lest you be concerned about the effect of a wrist watch to the whole body, let me tell you that it is estimated this would only be 40 millirems per year. I also have here a sample of uranium ore which gives a very major response on our counter.

Now, we would like to make the point right here that our instruments for detecting radiation are very good and we are, therefore, able to detect its presence when the amount is so small that it is many, many times below the harmful level.

RADIATION LEVELS

I have already made the point that radiation exists or has existed in the world at an average level of 160 millirems per year for many, many years and the human race has developed under these conditions. I would like to make an analogy—nitrogen oxide also occurs naturally. It is caused by sunlight causing a chemical reaction between the oxygen and nitrogen in the air. It is also caused by lightning flashes. The

amount of nitrogen oxide in the air varies considerably but a good average value for pure air is 4×10^{-9}, or four parts per billion. Nitrogen oxide is a very poisonous chemical, but obviously not harmful in these low concentrations. In cities due to industrial gases and particularly due to automobile exhausts the concentration goes up to as high as one part per million. Now the maximum allowable concentration on the basis of exposure, for eight hours per day, is five parts per million and if the concentration gets to approximately five hundred parts per million, a few breaths of this gas will probably be deadly. Now I would like to make a comparison of nitrogen oxides with radiation:

Radiation versus a Toxic Gas

	NO or NO_2 parts in air breathed	Radiation r/day (8)
Background	4×10^{-9} (4)	4.50×10^{-4}
In cities	$0.2\text{–}1. \times 10^{-6}$ (5)	—
Background and fallout		4.52×10^{-4}
Tolerance level for populations		4×10^{-3}
Tolerance for occupational 8-hr. exposure	5×10^{-6} (6)	6×10^{-2}
Lethal	500×10^{-6} (7)	600

Making comparisons like this is full of pitfalls because, strictly speaking, a chemical and its effect are not comparable with radiation and its effect. However, within broad limits the comparison is a valid one and brings out several very interesting points. In the case of the chemical, the acceptable tolerance for occupational exposure is about 1000 times the background but only about five to twenty-five times the values for polluted air. Whereas, in the case of radiation, the tolerance level for population is only about ten times background and for occupational exposures 150 times background. The comparison between occupational tolerance and lethal levels is 100 times in the case of nitrogen oxides but 10,000 times in the case of radiation. Let me point out, however, that in the case of nitrogen oxides if the lethal concentration is breathed for a very short time and if then proper medical measures are taken promptly, we probably can save all of those exposed with little or no permanent effect. However, in the case of exposure to lethal radiation levels, no matter how promptly or skillfully we act the best we could do might be to double the chances of survival and then all would agree that permanent damage has been done. Another point which is significant here is that because our instruments are so good we are able to detect radiation in levels which might be harmful, that is somewhat above tolerance, promptly and accurately. We might be much better in the case of chemical toxicity if we had instruments which were equally effective.

It is the generally held concept that any radiation is harmful. With regard to somatic effects, our information is not adequate for us to be

sure that exposure to very low levels of radiation shortens life or impairs the health or resistance to disease of individuals. There is general agreement, however, that continued exposure to radiation of moderately high levels, for example 100 roentgens per year, would definitely shorten life and reduce resistance to disease.

Any radiation is considered to be harmful from a genetic point of view. The geneticists apparently believe that radiation definitely causes mutations and the number of mutations caused is proportional to the amount of radiation received by the genes prior to the birth of the individual. It is also generally held by the geneticists that the vast majority of mutations are bad. There is a point, however, which should be emphasized at this time. The geneticists generally concede that of the presently occurring mutations probably a large fraction are due to causes other than radiation. These other causes are not well understood and work is under way to learn more about it. To be realistic, in balancing the possible genetic harm due to small increases in radiation levels against the need for weapon testing and the desire to use atomic energy in our economy, we need a better understanding of the possible genetic effects of our food, water, and industrial environment. The argument that the vast majority of mutations are harmful is based on the thesis that man is adjusted to his environment. It is worth commenting that our environment is presently undergoing some rather drastic changes.

This whole subject of the effect of radiation, both somatic and genetic, is extremely complicated. I would like to call your attention to hearings, "The Nature of Radioactive Fallout and its Effects on Man," which were conducted by the Joint Committee on Atomic Energy, May 27 to June 7, 1957. There is a tremendous amount of information on this exceedingly important and interesting subject contained in the record of these hearings. For those of you who are interested, I recommend that you get and read this material. There is also a short summary of it for those who merely want to get the highlights. I believe that our Congress and those participating in this hearing should be complimented for the excellent presentation.

These hearings covered the whole gamut of radiation levels from the naturally occurring background, through the comparatively innocuous levels which result from the atomic energy industry, to the very high levels which would occur in a nuclear war. It was brought out that ". . . multi weapon attacks (say 200 to 300 bombs of megaton size, 2500 to 3000 megatons total), can blanket half or more of the continental United States with lethal (death producing) or near-lethal radiation levels from local fallout alone. This, of course, could be applied to any nation. In addition to the acute effects in survivors as described above, chronic effects and genetic effects involving existing and future generations can occur . . ." (9). Although it is not my purpose to discuss nuclear war, I feel it is my duty to remind you again

that the effects are so staggering that we must find ways to avoid nuclear war if man is to continue to exist.

HANDLING RADIOACTIVITY

At the very beginning of this talk I made the point that we had been handling radioactive materials and had been running reactors for thirteen years with a phenomenally good safety record. This record is due to the fact that at the very start of the Manhattan project it was the determined policy to take all necessary precautions to avoid harm to the workers or the public. A series of publications has been put out by the government telling how to handle radioactive materials safely. The National Bureau of Standards Handbook series gives excellent guidance. The Radioisotopes Group which was established in the AEC in 1946 has also aided in guiding people in handling radioisotopes safely. On January 29, 1957, the Atomic Energy Commission issued regulations, Part 20, "Standards for Protection Against Radiation," and an amendment May 9, 1957. Through the divisions of Civilian Application and Inspection, the Atomic Energy Commission controls the use of radioactive material, which is a by-product of the atomic energy industry, making sure that it is handled in such a way that the workers and the public will not be in danger. There have been a few incidents in which the regulations have been violated and exposures slightly over the accepted tolerances have occurred. These should not happen.

Atomic energy installations, such as reactors, fuel element plants, and perhaps future chemical processing plants are increasing in number since private ownership was made possible by the amendment to the Atomic Energy Act of 1954. These installations must be reviewed to make sure that they are adequately designed, constructed, and operated so that they will pose no significant hazard to employees or public. The regulations, Part 50, describe the method of obtaining construction permits or licenses for these nuclear installations (facilities). In spite of the fact that no one has been injured by a nuclear reactor in regular operation these machines store enormous quantities of highly radioactive material. The nuclear reaction has the property of increasing in power level thousands of times in a fraction of a second. No other energy source has this potential. Accordingly, if a nuclear reactor should get out of hand, have a nuclear runaway as we say, it could melt or vaporize itself. By this means the radioactive fission products could be dispersed from the core. The nuclear reaction has another attribute which must be considered. Even though the nuclear reaction is shut off, heat continues to be generated in very significant amounts. This after-heat must be removed by maintaining the cooling for some time in order to avoid the melting of the reactor core. The basic problem is to keep the fission products contained and behind shielding walls so that the amount which can possibly escape is exceedingly microscopic and to prevent the gamma rays from them shining on any living

matter. Let us go back to our analogy with nitrogen oxides. You will remember that these materials are very toxic as chemicals go. Yet these chemicals are made at the rate of approximately forty tons a day in numerous plants throughout the country for the manufacture of nitric acid, an important industrial and agricultural chemical. Because we know how to build and operate these plants we do not worry about the problem and we operate the plants with no significant danger to the employees or the public. I believe we can learn to contain fission products also with no significant danger.

Because radioactivity and nuclear reactors are relatively new in our industrial experience, because the fission products are many times more toxic than any chemical, and because the industry has grown very rapidly (for example we are considering reactors making 500 grams or more per day of radioactive fission products) it was felt necessary that a different approach be taken to the hazard problem. This is to review the project and attempt to foresee the accidents which might happen. As early as 1948, a Reactor Safeguard Committee was formed. This Committee and its successors have been giving advice to the Atomic Energy Commission on the hazards of nuclear reactors up to the present time. On September 2, 1957, by an act of Congress, the Advisory Committee on Reactor Safeguards was made a statutory Committee and given the responsibility to advise the Commission on the safety of proposed or existing reactor facilities and to review and make public reports on privately owned testing and power reactors. A staff has been set up within the Atomic Energy Commission to evaluate the possible hazards of nuclear installations on a full-time basis. Thus, a machinery has been set up for reviewing proposed nuclear installations before they operate to be sure, as far as possible, that they will not result in a significant hazard to the employees or the public. As yet, there are no standards requiring reactors to be built in any special way. The industry is too new for that. The applicant for a license is responsible for the design, construction, and operation of the facility. The prime responsibility for the protection of the employees and public is his, as it should be. The Advisory Committee on Reactor Safeguards and the Atomic Energy Commission make sure that the job is done properly so that there is no unreasonable hazard.

In all of these facilities there are multiple barriers against the escape of radioactivity.

CONCLUSION

Using the first full-scale nuclear power plant in this country, I have tried[3] to show you the precautions which are taken and the barriers which are erected to prevent significant amounts of radioactive material from escaping and getting into the environment. However, there will be a certain amount of radioactivity that will get out. On the basis of

[3] EDITOR'S NOTE: Dr. McCullough used slides to explain the operation of the nuclear power plant at Shippingport, Pa.

our experience with other reactors, and particularly in view of the competence which has been shown in the design and construction of this one, and the care which will be taken in its operation, we feel certain that the amount of radioactivity released either to the ground, to the river, or to the air will be well below the level specified in Part 20 of the regulations. Since these levels in Part 20 are set on the basis of no injuries, the public can be satisfied that as long as the system that we have set up to protect them persists they have no real worries. Remember we are *NOT* promising that there will be no release of radioactivity, neither are we promising that there will be no accidents. We are saying that we have taken every reasonable precaution and perhaps even more to protect them against significant hazards.

Let me review briefly the fission reaction and the possible benefits it may bring to mankind. Notice that the reaction has the following characteristics:

1. A great deal of energy is released for a very small amount of fuel.
2. The reaction does not require any other ingredients, that is, no air.
3. The reaction can produce heat of substantially unlimited temperature (millions of degrees).
4. The fission produces more neutrons than are required to carry on the chain reaction (approximately 1½ extra).
5. The reaction is attended by intense radiation, betas and gammas, as well as neutrons.
6. The reaction produces fission products which are in themselves intense sources of beta and gamma radiation.

Approximately one gram of fissionable material is produced each day by a reactor producing a steady output of 1 megawatt of heat and the same amount of fission products result.

Let us see in what ways we are using these characteristics of the fission reaction:

1 and 2. The characteristics one and two—a lot of energy for little and no air required—are being applied very effectively to our nuclear powered submarines, the *Nautilus*, *Seawolf*, and *Skate*. For the first time we are able to operate a true submarine which can cruise indefinitely beneath the surface of the ocean.

3. In no case are we utilizing the third characteristic of the reaction —the unlimited temperature—for peaceful purposes. As a matter of fact, our power reactors are producing steam at considerably lower temperatures than are customary in our modern coal burning power stations. This is simply because we have not yet learned to handle the high temperatures in a nuclear reactor.

4. Neutrons permit us to realize the dream of the alchemist—making one element into another. But the only use we are making of this

is in the production of fissionable material for nuclear weapons and to a very minor extent for the production of radioisotopes.

5. The intense radiation field generated during the fission reaction is a nuisance today. We must provide thick shields to protect the public.
6. A very small part of the fission products being produced in this country today find uses. The uses to which they are put are important. Dr. Libby has estimated that they are saving upward of 200 million dollars a year. We have every reason to believe that their use will increase.

With the unique properties that atomic energy offers us we can look forward to an ever increasing use. Certainly the high temperatures available will find applications in metallurgy and chemical processing. The transmutation of elements for peaceful purposes certainly should be considered and finally as our supply of gas, oil, and coal becomes increasingly expensive and inadequate for our increasing needs for power, we will be forced to turn to atomic energy to supply our power needs.

We have learned to handle hazardous chemicals on a large scale. The chemical industry is proud of its safety record, which is about twice as good as the average of all industry. The atomic energy industry today has demonstrated that it, too, can have a good safety record. Currently, it is better than three times as good as the average of all industry. With the system we have set up for reviewing and controlling radioactivity, we believe that we can confidently expect to use atomic energy and still live with safe atoms.

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UNSTABLE LINEAR SYSTEMS AND THE MINIMUM PHASE CONDITION

BY

PAUL E. PFEIFFER [1]

INTRODUCTION

The well-known work of H. W. Bode on the mathematical characteristics of linear, time-invariant electrical transmission systems (1)[2] has proved to be a very important contribution to the fundamental theory of such systems. One of the keys to the development of this theory was the identification of and recognition of the significance of those systems described by minimum phase transfer functions, with their unique gain-phase relationships at real frequencies.

Bode has shown that any *stable*, linear, lumped-constant system may be represented by a transfer function which may be factored into a minimum phase transfer function times a number of factors, each of which is characterized by a combination of constant magnitude and lagging phase at real frequencies. These stable transfer functions, he shows, may be realized (theoretically at least) by passive electrical networks in cascade with an ideal flat voltage amplifier. Bode then remarks (1, p. 245) that this theorem leads one to suspect that any transfer function which has a radically different character from those of ordinary experience must represent an unstable system.

It is the purpose of this paper to examine explicitly the relationships between transfer functions for unstable systems and the class of minimum phase transfer functions. The present treatment will be limited to the case of rational transfer functions, although certain generalizations are at once obvious.

THE MINIMUM PHASE CONDITION

The term transfer function will be used to mean the ratio of the Laplace transform of the normal response of a system to the Laplace transform of the single input causing the response. By normal response is meant the response when the initial conditions are all zero; that is, when the system is initially "at rest." Since the basic work on the topic of minimum phase functions is that of Bode (1, cf. sections 11.5–11.8), some differences in notation should be pointed out to facilitate reference. Bode uses the term "transfer immittance" as a special case of what is here called the transfer function, with the difference that

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

he defines his functions in terms of input over output, whereas here the output over the input is used. Thus, the terms "phase lead" and "phase lag" must be interchanged as well as the terms "poles" and "zeroes." Bode's functions are special cases in that they are limited to functions describing the behavior of four-terminal networks having constant resistance characteristic impedance. The importance of the characteristic impedance termination is that combinations of elements in cascade have transfer functions obtained by multiplying the transfer functions of the individual units. This product law of combination, however, holds in other important cases, notably the case of non-interacting sections in cascade.

For simplicity of exposition, the present discussion shall be limited to the case of transfer functions $G(s)$ which are rational functions. This means that they are reducible to ratios of polynomials in the complex variable $s = \sigma + j\omega$. It is well-known that they must have real coefficients, so that they obey the conjugate law which states $G(\bar{s}) = \overline{G(s)}$. The bar over a complex number or a complex valued function indicates the complex conjugate of that number or function. The limitation to rational functions, imposed here, is not serious, for it covers practically all linear systems commonly studied.

A *minimum phase transfer function* $G(s)$ is one which has no poles or zeroes at points s for which the real part σ is positive. Except for the possible difficulties of poles at real frequencies ($s = j\omega$), the minimum phase condition implies system stability. It is assumed that poles on the real frequency axis are simple, in which case they do not affect stability. The development below will show the aptness of the term "minimum phase."

PHASE REDUCTION OF STABLE TRANSFER FUNCTIONS; ALL-PASS FUNCTIONS

A basic procedure in dealing with non-minimum phase functions is that which Bode calls *phase reduction*. This involves a mathematical trick which has a useful physical interpretation. Since the procedure is of wide interest and leads to the principal observation of this paper, it will be well to illustrate the mathematical procedure and discuss its meaning.

Suppose a transfer function of a stable system has zeroes at $s = a$, positive and real, and at $s = \alpha + j\beta$, with α positive. The transfer function may be written $G(s) = G_1(s)(s - a)[(s - \alpha)^2 + \beta^2]$, where $G_1(s)$ is minimum phase. Now the phase-reduction process involves replacing these zeroes in the right half-plane by their negatives as follows:

$$G(s) = G_0(s)\left[\frac{s - a}{s + a}\right]\left[\frac{(s - \alpha)^2 + \beta^2}{(s + \alpha)^2 + \beta^2}\right] \tag{1}$$

where

$$G_0(s) = G_1(s)(s + a)[(s + \alpha)^2 + \beta^2]. \tag{2}$$

The function $G_0(s)$ is the minimum phase function obtained from $G(s)$ by replacing the zeroes in the right half-plane by their negatives.

Consider the function

$$G_{a1}(s) = \frac{s - a}{s + a}. \tag{3}$$

At real frequencies (that is, for $s = j\omega$) this becomes

$$G_{a1}(j\omega) = -\frac{a - j\omega}{a + j\omega}. \tag{4}$$

Inspection shows that $|G_{a1}(j\omega)| \equiv 1$ and ang $G_{a1}(j\omega) \leq 0$ and is decreasing with increasing ω. Similar study of the function

$$G_{a2}(s) = \frac{(s - \alpha)^2 + \beta^2}{(s + \alpha)^2 + \beta^2} \tag{5}$$

shows it to have constant gain and lagging phase at real frequencies. Such transfer functions are called all-pass functions. They are non-minimum phase.

It is readily seen that the above process is quite general, and that any stable non-minimum phase function $G(s)$ can be expressed as a minimum phase function $G_0(s)$ multiplied by a finite number of all-pass functions, each being one of the two types considered above. Because the amplitude of the all-pass functions is unity for real frequencies, it follows that $|G(j\omega)| \equiv |G_0(j\omega)|$. The angle of lag of $G_0(j\omega)$ is, however, less than the angle of lag of $G(j\omega)$; hence, the term "phase reduction" is used for the above process. Examples of all-pass networks are well known. In the case of noninteracting circuits, any of the constant-amplitude phase shifting networks is of this class.

ALL-PASS FUNCTIONS AND UNSTABLE TRANSFER FUNCTIONS

An unstable transfer function $G(s)$ is characterized by the presence of poles in the half-plane for which the real part of s is positive. Mathematically it is of the same form as the non-minimum phase transfer function, except that the factors $(s - a)$ and $[(s - \alpha)^2 + \beta^2]$ are in the denominator. An exactly similar process of moving poles from the right half-plane to their negatives in the left half-plane may be carried out. Thus, for one real positive pole and a pair of conjugate complex poles in the right half-plane the following holds:

$$G(s) = G_0(s)\left[\frac{s + a}{s - a}\right]\left[\frac{(s + \alpha)^2 + \beta^2}{(s - \alpha)^2 + \beta^2}\right], \tag{6}$$

where $G_0(s)$ is *stable* and has the *same amplitude at real frequencies* as does $G(s)$. The transfer functions

$$G_{a1}'(s) = \frac{s + a}{s - a} \tag{7}$$

and

$$G_{a2}'(s) = \frac{(s + \alpha)^2 + \beta^2}{(s - \alpha)^2 + \beta^2} \tag{8}$$

are reciprocals of the stable all-pass functions considered above. Hence, they are *all-pass with phase lead*. Since, however, they have *poles* in the right half-plane, they are *unstable*.

Obvious generalization of the above procedure to any number of poles and zeroes in the right half-plane shows that every rational transfer function $G(s)$ may be written as a minimum phase transfer function $G_0(s)$ times a finite number of all-pass, phase lag (stable) functions and a finite number of all-pass, phase lead (unstable) functions, with the condition that $|G_0(j\omega)| \equiv |G(j\omega)|$. Since the special case $G(s) \equiv 1$ (or -1) is a minimum phase all-pass function, and since the gain-phase relationship of a minimum phase function is unique, it follows that the only rational all-pass functions are of the types considered above.

The theoretical picture in regard to gain-phase relationships in rational transfer functions may be summarized as follows.

(*i*) The only *stable all-pass functions* other than constants are characterized by *increasing phase lag* at real frequencies.

(*ii*) If $G(s)$ is rational and $G(j\omega)$ is prescribed, then the "minimum phase lag" of Bode's theory is the minimum phase shift in the direction of lag for which the system is stable.

(*iii*) A non-minimum phase function differs from a minimum phase function only by all-pass functions. In the case of instability, the variation is by one or more phase lead, all-pass functions.

THE STABILITY CRITERION

The previous treatment shows that the amplitude (gain) characteristics of unstable transfer functions do not differ from the gain characteristics of stable transfer functions. The difference lies in the related phase characteristics. If phase and gain (in db) are plotted against frequency to a logarithmic scale, the well-known theorems of Bode (see reference (1) or any of several books on servomechanisms) make it possible to visualize easily the minimum phase characteristics corresponding to the gain characteristic. If necessary, the phase characteristic can be determined graphically with considerable accuracy. Any radical departure from these characteristics is easily detected. Not

only can the departure from the minimum phase condition due to zeroes in the right half-plane be detected, but the presence of poles in the right half-plane can also be seen. The only situations in which this might be difficult is the highly improbable (though possible in principle) situation in which a zero and a pole are in close proximity. In the usual case of instability, $G(s)$ would have no zeroes in the right half-plane, so that the instability would be indicated by phase lag *less* than the "minimum phase lag" by the amount of the phase lead of one or more of the all-pass functions of the type in Eq. 7 or 8.

The example in Fig. 1 will serve to illustrate the nature of the test.

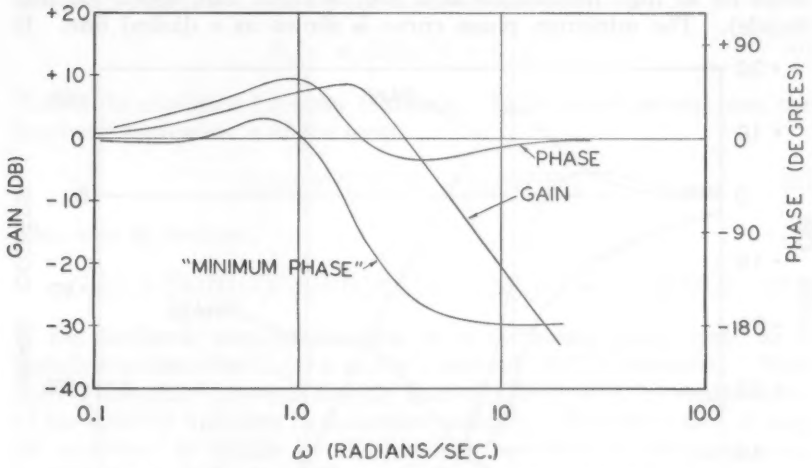

FIG. 1. Comparison of phase in a stable and unstable system having the same real frequency gain characteristics.

The transfer function is as follows:

$$G(s) = \frac{(T_1 s + 1)}{(T_2 s - 1)[(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1]}.$$

The minimum phase curve is shown as a dashed line. It rises slightly because of the peaking of the gain, then drops toward the 180° high frequency asymptote predicted by Bode's theory. The actual phase curve, shown as a solid line, shows decidedly less than the "minimum phase lag." In this discussion, the minus one d-c. gain of the system is not considered as 180° phase shift. Multiplication by minus one, which in an electrical system corresponds to reversal of a pair of terminals, is fundamentally different from a change of phase shift with frequency of 180°.

A somewhat more complicated example is shown in Fig. 2. It has

a zero at $s = \omega_0/2$ and two pairs of conjugate complex poles. One pair is in the left half-plane with an undamped natural radian frequency ω_0 and relative damping $\zeta = \frac{1}{4}$. The other pair, associated with the instability of the system, is in the right half-plane, with an undamped natural radian frequency of $2\omega_0$ and damping ratio $\zeta = \frac{1}{2}$. The transfer function is:

$$G(s) = \frac{(2s/\omega_0 - 1)}{[(s/\omega_0)^2 + (1/2)(s/\omega_0) + 1][s/2\omega_0)^2 - (s/2\omega_0) + 1]}.$$

The gain curve rises about 14 db in the neighborhood of $\omega = \omega_0$ and drops off at high frequencies at a rate of three unit slopes (60 db/decade). The minimum phase curve is shown as a dashed line. It

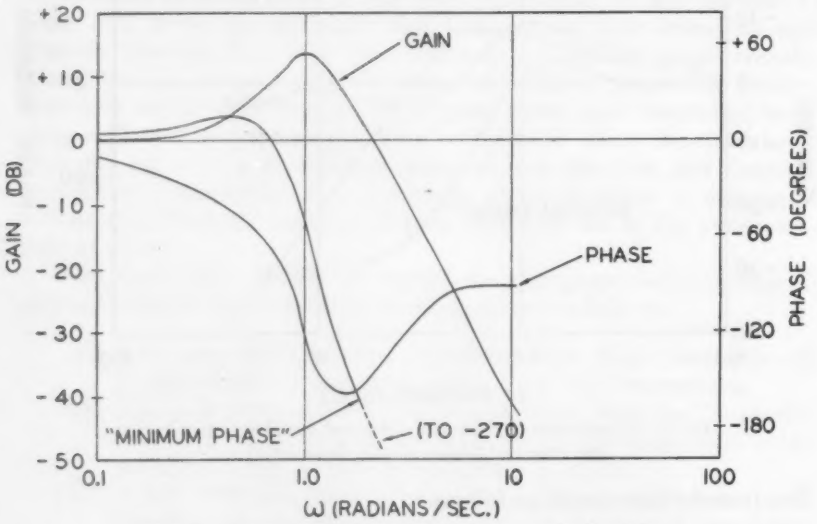

FIG. 2. A second example of an unstable transfer function.

rises slightly in a frequency range below $\omega = \omega_0$ because of the peaking of the gain curve, then drops rather sharply toward its high frequency asymptotic value of $-270°$.

The phase curve for the transfer function as described, however, differs markedly from the minimum phase curve. It first dips sharply because of the extra phase lag due to the zero at $s = \omega_0/2$ (a non-minimum phase condition). This trend is rapidly arrested by the extra 360° phase lead contributed by the poles in the right-hand plane (indicating instability). The net result is that the high frequency phase asymptote is $-90°$ rather than the $-270°$ associated with the minimum phase condition. The condition of instability is thus clearly indicated in the phase curve.

The examples discussed in the preceding paragraphs illustrate the nature of the stability criterion when system gain-phase curves are known. In most systems where a stability criterion is needed, it is desired to determine closed loop stability from considerations of open loop characteristics. This is the approach of the Nyquist theory and its various equivalents and modifications. The present test requires the closed loop characteristics. When logarithmic units are used (gain in db, frequency to log scale), as is usually the case, conversion from open loop to closed loop characteristics is relatively easy and rapid. The well known Nichols' chart provides for conversion from the open loop gain-frequency function $G(j\omega)$ by the transformation

$$G_c(s) = G(s)/[1 + G(s)]. \tag{9}$$

This is the condition for unity feedback. In the more general case, the feedback expression is of the form

$$G_c(s) = G(s)/[1 + G(s)H(s)]. \tag{10}$$

This may be written

$$G_c(s) = [1/H(s)]G(s)H(s)/[1 + G(s)H(s)] = G_c'(s)/H(s). \tag{11}$$

If the feedback transfer function is of minimum phase type, as is usually the case, then $G_c(s)$ is stable if and only if $G_c'(s)$ is stable. Since the Nichols chart converts directly from $G(s)H(s)$ to $G_c'(s)$, examination of the latter is sufficient to determine stability. In other cases, it may be necessary to divide by $H(s)$, which amounts to subtraction on logarithmic coordinates. The added complication is ordinarily not serious, especially since the feedback function $H(s)$ is usually simpler than $G(s)$ and is usually represented quite accurately by its analytical expression. All of the usual tricks of approximate handling familiar to the servomechanism designer (see any of a number of readily available servo books) may be employed to expedite handling of the gain and phase expressions.

PHYSICAL REALIZATION OF AN ALL-PASS PHASE-LEAD SYSTEM

The theoretical role played by the all-pass phase-lead system raises the questions: (1) Can such systems be built physically? and (2) If they can be realized physically, can they be stabilized so that the gain-phase relationship may be observed? Both of these questions have been answered in the affirmative in the case of the first order function of Eq. 7. A group of graduate students succeeded in synthesizing and observing these characteristics in a laboratory assignment carried out under the direction of the author. The principal ideas and techniques are outlined below.

Suppose it is possible to obtain a network (necessarily active) with high input impedance and low output impedance with voltage transfer function

$$G^0(s) = \pm K(s + a)/(s - a). \tag{12}$$

How could it be stabilized and how could the results be observed? Now it is known that systems exist which are open-loop unstable but closed-loop stable. It seemed reasonable to investigate the possibility of stabilizing the all-pass phase-lead system by including it in a feedback loop. One of the simplest feedback configurations is that shown in Fig. 3. With the notation of that figure and the assumption of zero

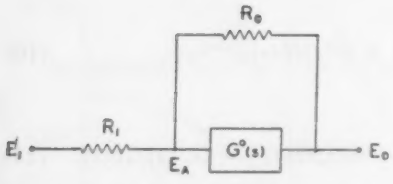

FIG. 3. A feedback system for stabilizing an all-pass phase lead transfer function $G^0(s)$.

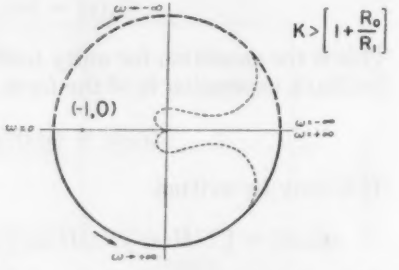

FIG. 4. The transfer locus $K[R_1/(R_1 + R_0)]$ $[(s + a)/(s - a)]$.

input current and zero output impedance for the all-pass system, straightforward analysis shows the gain of the system to be expressible in the form

$$\frac{E_0(s)}{E_1(s)} = -\frac{R_0}{R_1} \cdot \frac{-R_1G^0(s)/(R_1 + R_0)}{1 + [-R_1G^0(s)/(R_1 + R_0)]}. \tag{13}$$

Expression 13 is in a form to which the Nyquist stability criterion applies. Because G^0 has a pole in the right-hand plane, however, the general form of the Nyquist criterion must be used. For the special case above, the number of encirclements of the critical point $(-1, 0)$ by the transfer locus $-R_1G^0(j\omega)/(R_1 + R_0)$ must be plus one. Encirclements are positive if the critical point is encircled in a counterclockwise direction as ω goes from 0^+ to $+\infty$ to $-\infty$ to 0^-. Figure 3 shows the transfer locus for the case that the minus sign in expression 12 is used, so that the transfer locus becomes $[KR_1/(R_1 + R_0)]$ $\times [(s + a)/(s - a)]$. If $K > 1 + R_0/R_1$ then the encirclement situation is that required for stability, as is indicated in Fig. 4. If G^0 were a truly all-pass system, the sign in Eq. 12 would be immaterial, since a change of sign would merely rotate the transfer locus by 180°. This obviously would not affect the circular locus. However, it is inevitable

that the gain of the system must drop off at very high frequencies. This condition is indicated in Fig. 4 by the dotted curve bending back toward the origin. Thus in an actual system the sign is important. In the stable case, the system is actually conditionally stable, since stability depends upon having sufficient loop gain to provide the needed encirclement.

Once the system has been stabilized, the observation of the gain-phase relationship is quite easy. The over-all system of Fig. 3 is excited sinusoidally at E_1. The sinusoidal voltages E_0 and E_A are observed. The relation of the latter two gives the desired gain and phase quantities for the all-pass system G^0.

The question concerning stabilizing and observing the behavior of the all-pass phase-lead system has been answered. There remains the question of synthesizing the system. This is accomplished by simply inverting an all-pass phase-lag transfer function by means of operational amplifier techniques. The term operational amplifier is borrowed from analog computer terminology, and refers to a high gain d-c. amplifier utilized in certain standard feedback arrangements.

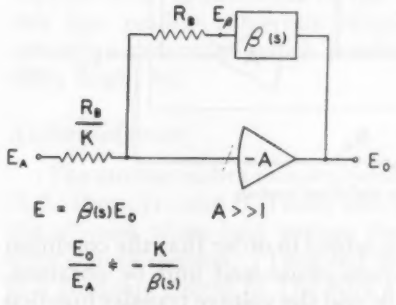

FIG. 5. A feedback system for inverting transfer function $\beta(s)$.

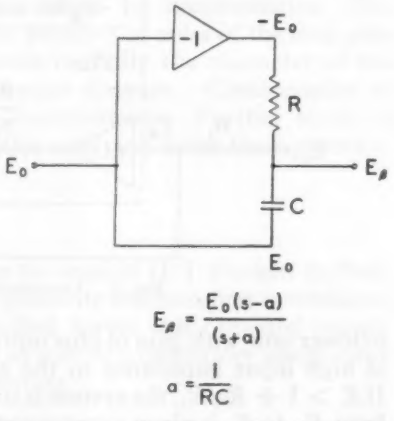

FIG. 6. A practical all-pass phase-lag circuit.

The circuit of Fig. 5 provides a means of inverting a voltage transfer function $\beta(s)$. If $E_\beta = E_0\beta(s)$ and if A is very large, then standard linear theory shows that

$$E_0/E_A = -K/\beta(s) \quad \text{(approx.)} \tag{14}$$

to a high degree of approximation.

A simple means of realizing $\beta(s)$ as the transfer function for an all-pass phase-lag system is shown in Fig. 6. An operational amplifier

with two resistors may be used to provide a gain of -1. The voltage at capacitor C is thus $+E_0$ and at the end of resistor R is $-E_0$. The open circuit voltage E_β is readily shown by use of superposition to be given by

$$E_\beta = -E_0(s-a)/(s+a), \quad \text{where} \quad a = 1/RC. \tag{15}$$

If in the circuit of Fig. 5 the value of resistor R_B is very much larger than R, then the loading effect is negligible, and the circuit of Fig. 6 provides the desired $\beta(s)$. The elements are combined as shown in Fig. 7 to provide the stabilized system. In the circuit of Fig. 7 a

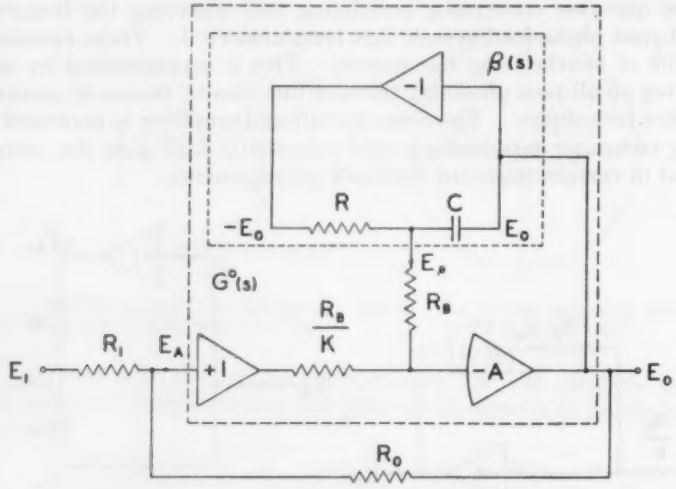

FIG. 7. The complete stabilized system.

follower unit with gain of plus unity is added in order that the condition of high input impedance to the all-pass phase-lead unit be obtained. If $K > 1 + R_0/R_1$, the system is stable and the voltage transfer function from E_A to E_0 is given approximately by

$$E_A/E_0 = K(s+a)/(s-a), \quad \text{where} \quad a = 1/RC \tag{16}$$

as desired. Actual laboratory setups have verified the all-pass phase-lead characteristics. Measured values of phase shift correspond to theoretical values within expected experimental accuracies. The value of a was kept to the order of 100 (corresponding to a frequency of about 16 cycles per second). Thus, the essential behavior of the system lies in a range under 1000 cycles per second, and high frequency parasitic effects are negligible.

CONCLUSIONS

Unstable systems have been shown to differ from stable systems at real frequencies only in the phase characteristic. This provides a simple test for stability in many practical cases. Also, it has been shown that the unstable system is equivalent to a stable system in cascade with all-pass phase-lead elements. There are two types of the latter, one of first order and one of second order. The first order system has been synthesized, stabilized, and its behavior observed in the laboratory.

The question remains whether the all-pass phase-lead unit is more than an academic curiosity. One practical question has seemed to merit attention. The problem of stabilizing feedback systems often leads to the point that it would be desirable to provide leading phase compensation without the corresponding rise in gain with increasing frequency. This plus the fact that many open-loop unstable systems may be stabilized by feedback raises the question whether or not troublesome systems may be stabilized by phase-lead all-pass compensation networks. The question has been investigated but no definite results have been reached. There are indications that the possibilities are severely limited theoretically, to say nothing of limitations due to system complexity and of problems raised by nonlinearities. The theoretical difficulty arises in that the presence of poles of the loop gain function in the right hand plane alters radically the character of the encirclement requirements of the Nyquist diagram. Consideration of this fact readily rules out certain combinations. Further study is needed to determine which possibilities exist and what their practicability might be.

Acknowledgment

The author wishes to acknowledge the work of H. J. Harker, A. Nall, H. L. Pope, Jr., and B. Tasini who as graduate students first synthesized the all-pass phase-lead system described herein and obtained experimental data thereon. They did this as part of a laboratory assignment in a graduate course at The Rice Institute.

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(1) H. W. BODE, "Network Analysis and Feedback Amplifier Design," New York, D. Van Nostrand, 1945.

(2) N. BALABANIAN AND W. R. LEPAGE, "What is a Minimum-Phase Network?" *Communication & Electronics*, Jan., 1956, pp. 785–787.

Giant Radio Telescope.—A giant radio telescope that will follow the sun across the sky by day and catch signals from outer space by night will be built next June at The University of Michigan. Fred. T. Haddock, associate professor of astronomy and of electrical engineering, said the highly-sensitive instrument will be erected at Peach Mountain, 16 miles from Ann Arbor.

Standing over 100 ft. high, the instrument will have a solid "dish" reflector 85 ft. in diameter. The device will require 160 tons of steel and aluminum, 50 tons of lead counterweight, and 250 cu. yd. of concrete foundation.

In addition to being the second largest steerable radio telescope in the world, it will be one of the most precisely made, Professor Haddock pointed out.

"With this instrument," he said, "we will be able to receive radio waves only a few centimeters long, and to focus on only three per cent of the sun's surface at a time. This is fine detail, and will put us far ahead in the field."

The "dish" will be able to pick up the short wavelengths because of its precise construction. The surface will not vary more than a quarter inch from a true parabola. The larger diameter will allow U-M researchers to collect fainter signals, and to "look at" smaller portions of radio sources, he added, and the dish's solid aluminum surface will prevent the leakage of signals at all wavelengths.

The complete facility will cost about $300,000, Haddock said, with most of the money supplied by the Office of Naval Research.

The new telescope will join a 28-ft. unit placed in daily automatic operation at Peach Mountain in August. The smaller unit already has recorded some "new and unusual" signals from the sun.

Besides receiving solar signals, the newer instrument will hear the faint whispers from the moon, planets, galaxies millions of light years away, turbulent gas clouds, "radio stars," and other sources, some possibly beyond the range of the best optical telescopes. These signals are produced by the interaction of electrons with atomic particles or magnetic fields, and yield data about the regions in which they originate.

Two large gears will drive the U-M dish. A declination gear will tilt it north and south, while an hour angle gear will move it east to west to compensate for the earth's rotation. It will be possible for the unit to follow a sunspot or solar flare region all day, or to execute "elaborate scanning operations."

The telescope will be operated in winds as high as 45 miles an hour and will survive 120 mph winds.

The dish will work at wavelengths 5 to 10 times shorter than the world's largest "dish"—a 250-ft. unit in England—and will have three times the collecting area of any other in its class.

There are other types of radio telescopes which stretch out for hundreds of yards. They provide a high degree of resolution, but cannot be moved to follow radio sources or the sun for many hours at a time.

A VACUUM FURNACE FOR ELECTRICAL CONDUCTION STUDIES UP TO 2000° C.*

BY

W. E. DANFORTH [1] AND H. BLEECHER [2]

ABSTRACT

Constructional details and performance characteristics of a vacuum furnace, suitable for studies of properties of matter and operable to 2000° C., are given. By virtue of a relatively high impedance design, the unit may be operated on d.c. thereby avoiding troublesome a-c. pick-up in measuring circuits. The volume of the hot zone is approximately 3 cm. in diameter and 12 cm. long; power required for 2000° C. is approximately 6 kilowatts.

I. PRELIMINARY

The design of a vacuum furnace for the conduct of experiments on the properties of matter at high temperature introduces certain requirements which are not present when the unit is to be used solely for heat treatment of materials. Among them are uniformity of temperature and absence of a-c. pick-up in measuring circuits. The high temperature volume should therefore be reasonably large and the unit should be operable on d.c.

The simplest mechanical design for a furnace is attained if currents of the order of thousands of amperes are used for heating; the heating element can then be simply a thin-walled tube. But it is in certain ways inconvenient to use such high currents, particularly if d.c. must be used to avoid pick-up.

Thus for a number of reasons the type of vacuum furnace which has been developed in this laboratory is one in which the heating unit has relatively high resistance. The current-supply leads are not inconveniently massive and if d-c. operation is desired the source of power required is not difficult to obtain.

II. BRIEF DESCRIPTION

The main features of the present design are shown in Fig. 1. A coil of about 24 turns of 80-mil tungsten wire is mounted horizontally in coaxial molybdenum heat shields. The coil is suspended by 20 mil tungsten wire hangers which are themselves supported by alumina insulators. In this way the electrical insulation is removed from the high temperature region. In Fig. 1, 6 hangers are shown. Since making this drawing, experience has shown that sag free operation at

* Supported in part by the U. S. Navy, Bureau of Ships.

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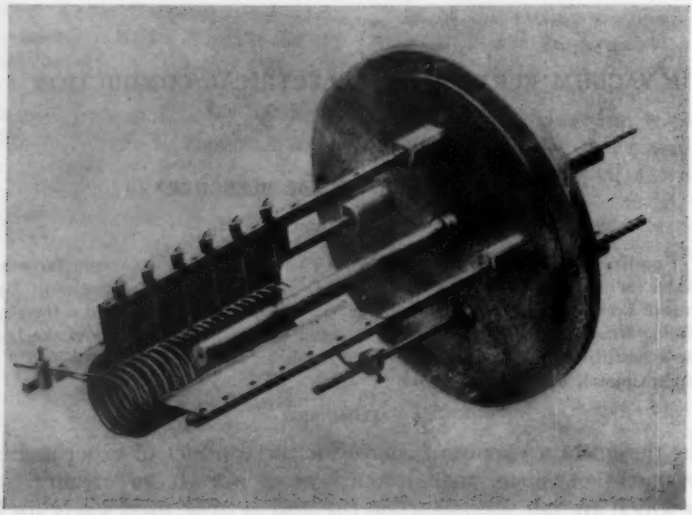

FIG. 1. Vacuum furnace for high temperature measurements. Cut away portion shows tantalum shield containing specimen being studied. In present design the number of hangers has been increased from 6 to 12. (Drawing by Mary Elizabeth Brandt.)

highest temperatures requires 12 hangers; the coil may be designed so that every other turn is supported. The choice of horizontal rather than vertical construction is dictated largely by the problem of supporting the high resistance heating unit. Figure 2 shows schematically the

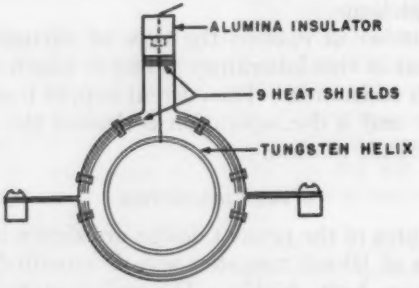

FIG. 2. End view of unit, showing arrangement of heat shields.

arrangement of coil, heat shields, and insulators. The upper portion of each insulator is intentionally made large to decrease, by radiation, its operating temperature.

Principal dimensions of the unit are the following: outside diameter of coil $1\frac{3}{8}$ in., length of coil and heat shields is 5 in., inside diameter of heat shields is $1\frac{3}{4}$ in. The three supporting bars are cold rolled steel,

the top one having a $\frac{3}{8}$ in. $\times$ $\frac{5}{16}$ in. cross section, and the side bars a $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. cross section. The material of the heat shields is 5-mil molybdenum sheet.

The broken away portion of the coil in Fig. 1 shows a coaxial tantalum tube which, at the present writing, bears a specimen whose conductivity at high temperature is being investigated. A hole in its front end provides a "nearly black" body for reasonably accurate optical pyrometry.

III. HEAT SHIELDING

In a vacuum furnace the only heat loss is by radiation (neglecting the relatively small loss by conduction in leads.) Control of heat loss is therefore not accomplished by "insulation" but by reflection from metal heat shields. By using a sufficiently large number of successive reflectors the heat loss through a given surface may be made as small as one pleases, a practical minimum being set by space limitations. Careful spacing of successive shields to avoid mutual contact is not necessary; physical contact between shields, even when loosely riveted together, occurs at relatively few points and actual heat flow by conduction is not large. Large areas of contact are avoided by making an array of punch marks on the individual laminations.

As shown in Fig. 2, the outermost of the heat shields provides mechanical support for the others; the additional reflecting surfaces are riveted inside. As shown in Fig. 3 the steel bar bearing the insulators

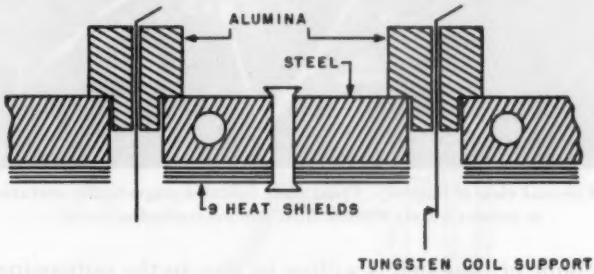

FIG. 3. Detail of insulators. The heat shields prevent the supporting steel bar from reaching excessive temperatures.

has heat shields riveted to its lower surface; these prevent the steel bar from reaching excessive temperatures. In each location a total of 9 reflecting surfaces is used.

IV. JACKET AND VACUUM SYSTEM

The jacket is shown in Fig. 4. As regards water cooling, we have experience both with double-walled construction and with copper tubing

soldered to a single-walled jacket; the latter was chosen as being both more positive and less likely to be a source of vacuum leaks. Coverage by the copper tubing must, however, be fairly complete. The tubing on the main body should be brought close to the end flanges. Cooling of the end plates is also essential. Vacuum seals are by rubber "O" rings.

The vacuum system consists of a Welch type 1405 fore pump, a type VMF 260 oil diffusion pump from Consolidated Vacuum Corporation with a type WB8-01 baffle from the same company. Ultimate vacuum for this system is about 1×10^{-6} mm. Hg when cold; the degree of vacuum at high operating temperature will be determined, of course,

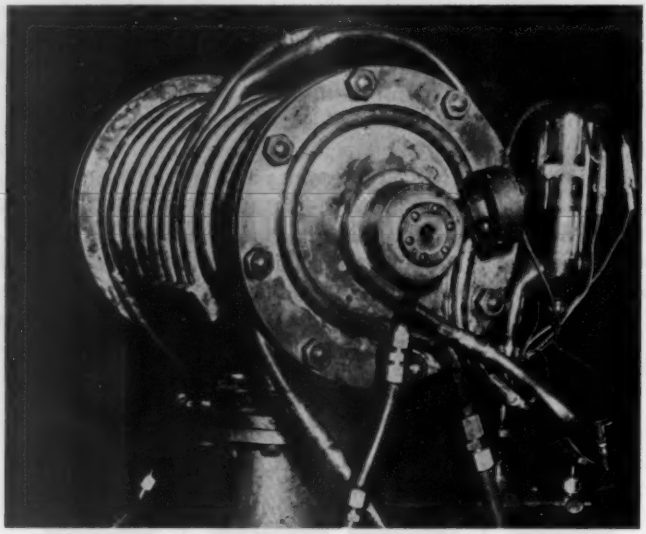

FIG. 4. External view of furnace. Front plate contains magnetically operated shutter to protect quartz window from any evaporated material.

by the amount of time one is willing to give to the outgassing process. With the furnace running continually, the same ultimate figure applies. In our present work, the unit is operated for long periods at 1300°C; opening only for change of specimen, and pressures less than 5×10^{-6} are the rule. Periods of operation at 2000°C. have not been for more than 5 or 6 hours at a time; pressures thus far experienced at this temperature are about 5×10^{-5} mm. We see no reason why the 1×10^{-6} ultimate figure would not also apply to 2000° operation.

Figure 5 gives a cross section of the back plate on which the furnace unit is mounted. Figure 6 is a detail of the shutter arrangement which is attached to the front plate as shown in the photograph. Use of the

shutter to protect the window from darkening is essential for accurate pyrometry.

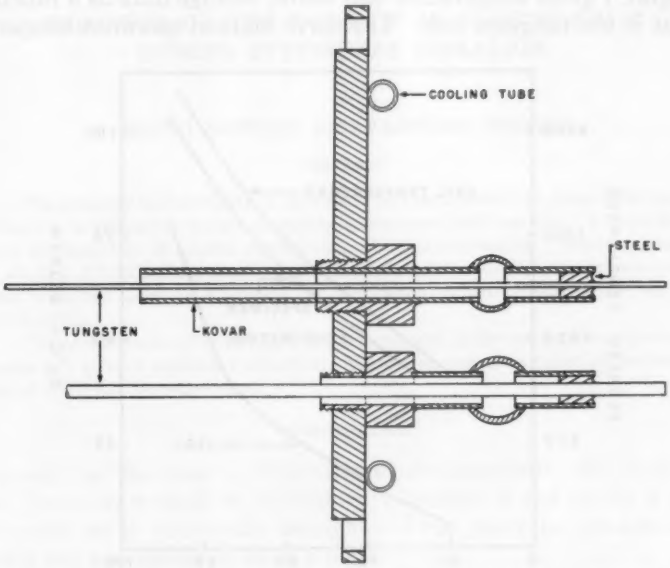

FIG. 5. Back plate, showing leads for heating current and for electrical measurement.

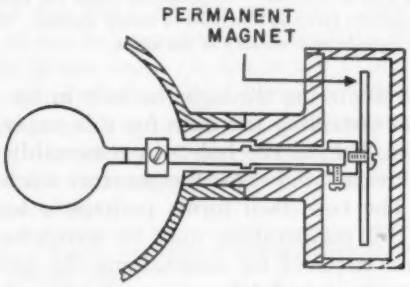

FIG. 6. Detail of magnetically operated shutter.

V. OPERATING CHARACTERISTICS

The furnace which we are at present using for conductivity studies is heated by a.c. while outgassing is proceeding and when left running overnight; it is switched to d.c. when actual measurements are being made. The heating unit described above is operated by two 3 KVA transformers with 220 V primary and 32 V secondary. The secondaries are connected in series and the primaries are operated in parallel from

a type 50B 7 KVA General Radio variac. For d-c. operation, the heating element is switched to a separately excited 75 V generator.

Figure 7 gives temperature and heater voltage data as a function of current in the tungsten coil. The curve marked specimen temperature

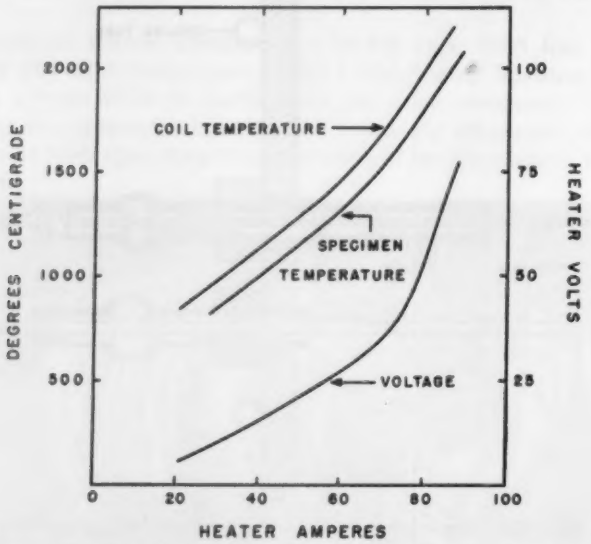

FIG. 7. Operating characteristics. Temperatures inside the specimen holder and temperature of the heater coil are given as functions of heater current. Heater voltage ordinate values are given on the right-hand boundary of the figure.

is obtained by pyrometering through the hole in the specimen mount. For the purpose of obtaining the data for this paper, a tantalum disc was mounted $\frac{1}{16}$ in. behind the hole. A reasonably good black body cavity is thereby created and these temperature readings must be close to true values. The coil itself forms perhaps a less perfect cavity; values given for coil temperature may be somewhat below the true values. The power required for maintaining the specimen at 2000°C. is seen to be approximately 6 kilowatts; a log-log plot shows that the fourth power relation between heater power and absolute temperature is followed. It is estimated that slightly more than half of the input power is radiated from the open ends of the heat shields. In other words, almost half of the power is transmitted through the large area of the heat shields; for higher temperature operation it might be advisable to increase their number, and to make the innermost shields of sheet tungsten to avoid evaporation.

BEHAVIOR OF THE NODE OF AN ELASTIC BEAM DURING STATIONARY VIBRATION

BY

SEIICHI HIGUCHI[1] AND KAZUKIYO IINUMA[2]

ABSTRACT

The property of immobility of the node of an elastic beam in a state of stationary vibration is utilized in designing certain instruments and machines[3] in accordance with the teachings of applied physics and vibration engineering. However, except in the case of infinitesimal displacement, the node does not always remain as a special fixed point but moves on a straight line or a closed curve according to its condition of vibration.

The present paper gives examples of the movement of the node and its neighboring points in a state of stationary vibration. Various photographs, taken at the time of forced vibration, are reproduced.

INTRODUCTION

Except for the case of infinitesimal displacement, the node of an elastic beam in a state of stationary vibration is not really a special fixed point as is commonly thought. This paper is concerned with showing the movement of such a node.

For instance, imagine a cantilever in a stationary state of forced vibration. To the clamped end of a straight phosphorbronze cantilever (750 mm. long, 0.5 mm. thick, 12 mm. wide) we apply frequency varying forced vibration at the half amplitude of 5 mm.[4] We mark previously the thickness side of the beam (it lies in the plane of vibration) with tiny white cream spots at intervals of 2 mm. With each increase of the frequency, a photograph was taken in a dark room after being exposed at least for more than one period of vibration and superposed. Figure 3 shows distinctly the various paths of movement of the drops of cream, for different frequencies.

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[1] Professor, Faculty of Engineering, Iwate University, Morioka, Japan; Professor Emeritus of Tohoku University, Sendai, Japan.

[2] Lecturer, Faculty of Engineering, Tohoku University, Sendai, Japan.

[3] Called "Onpen" in Japanese, a free-free bar which is supported by a thin metal rod at the two nodes corresponding to the fundamental mode of vibration is widely used as the frequency standard in the tele-photograph in Japan. This can be compared to several other kinds of beams including the tuning fork, since the dissipation of energy at the time of vibration of the free-free bar supported at its two nodes may probably be very minute, so that variation or fluctuation of the frequency may naturally be expected to be minute also. An example of "Onpen" is shown in Fig. 1.

[4] The experimental apparatus and method used here are almost the same as those described in a previous paper by one of the present authors, *viz.*, *Technol. Rep. Tohoku Imp. Univ.*, Vol. X, p. 12 (1932). The present apparatus is shown in Fig. 2.

MOVEMENT OF THE NODE AND ITS NEIGHBORING POINT

The several photographs in Fig. 3 were selected from those corresponding to the various frequencies to show: (*i*) a small amplitude without any node; (*iii*) with a node; (*iv* and *v*) with a node rising along the beam; (*vi*) occurrence of the second node at the base; and (*vi, vii* and *viii*) rising of the nodes with increase and decrease of the amplitude.

To illustrate the phenomenon more clearly, the paths of the node and its neighboring point shown in (*vi*) and (*vii*) of Fig. 3, are enlarged in Fig. 4. Figure 4 shows plainly that the node in (*vi*) makes a vertical

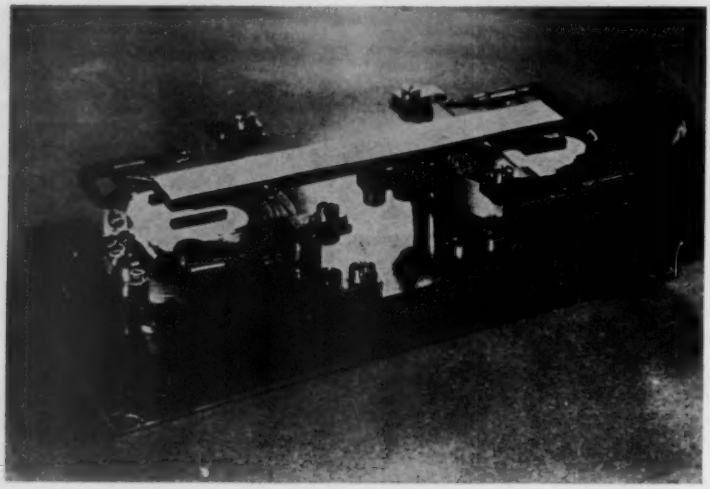


FIG. 1. "Onpen" in Japanese, or the tuning bar which is supported by the thin metal rod at the two nodes corresponding to the fundamental vibration of the free-free bar.

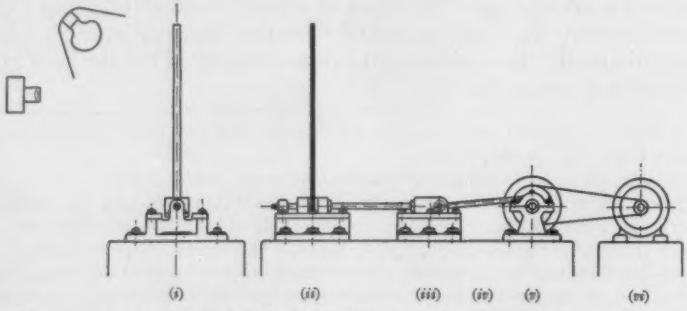
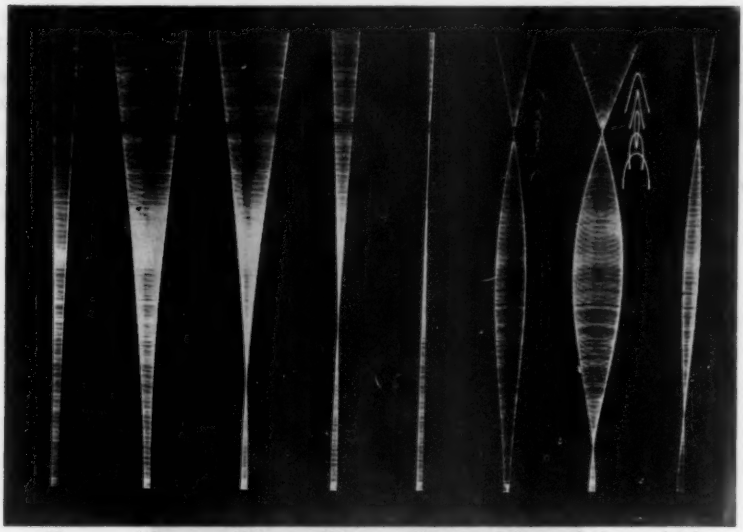


FIG. 2. Experimental apparatus for the forced vibration of an elastic beam, used in this paper.

(*i*) Test piece and oscillation table.
(*ii*) Side view of test piece and oscillation table.
(*iii*) Crosshead
(*iv*) Connecting rod
(*v*) Crank
(*vi*) Motor



(*i*) (*ii*) (*iii*) (*iv*) (*v*) (*vi*) (*vii*) (*viii*)

FIG. 3. Panoramic photograph obtained by starting with very slow motion and increasing the frequency sharply, after spotting the whole beam at intervals of 2 mm. The frequencies corresponding to the various cases are shown below. The natural frequency is 0.767 per second.

(*i*) 0.650	(*v*) 1.833
(*ii*) 0.717	(*vi*) 5.832
(*iii*) 0.800	(*vii*) 6.267
(*iv*) 0.917	(*viii*) 7.000

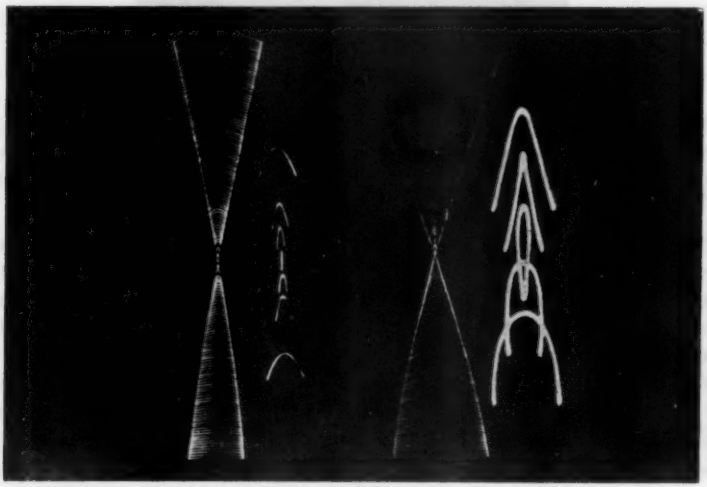


FIG. 4. Enlarged photographs of a part of the vibrating beam shown in (*vi*) and (*vii*) of Fig. 3.

rectilineal motion instead of "a point," and the first node in (*vii*) (the upper one) draws a closed curve.[5] When the frequency is further increased and the amplitude diminished as shown in (*viii*), the node again forms a straight line and if the amplitude becomes much smaller, it will be reduced to a primary point.

In Fig. 4, only the paths of the node and its very near points are shown. By taking a wider range of the part of a beam, the photographs in Fig. 5(*i* and *ii*) are obtained. The numbers in Fig. 5 represent the

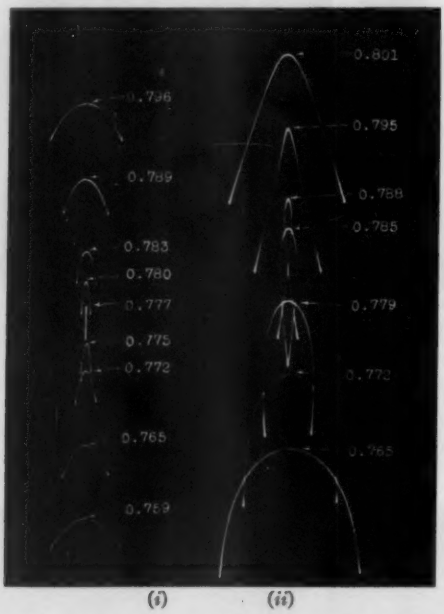


FIG. 5. (*i*) and (*ii*) show the movement of the node and its neighboring points, previously shown in Fig. 3, (*vi*) and (*vii*), respectively. Numbers in the drawing represent the ratio of the spot from the clamped end of the beam to the total length of it. The two drawings are equally magnified.

ratio of the height of the point in question to the total length of the beam. These photographs seem to tell us many things, for example: (*i*) the larger or the smaller the amplitude of the vibrating beam, the larger or the smaller the area of the closed curve described by the node; and (*ii*) if we connect lengthwise the extreme points of the amplitude of every point of a beam on the picture, we will find that some points situated just below the intersecting point of the two curves obtained here (this

[5] If the given beam becomes long, enlarging the amplitude of the clamped end, the closed curve of the node mentioned here may easily be observed even if under the condition of one node. Further, the highest point of the above mentioned straight line or closed curve corresponds to the theoretical true node, respectively.

corresponds to the lowest position of the node under the given condition) move in the outside range of the region bounded by the curves in question.

Examples of damping behavior of the area of the closed curve due to the node with the frequency are given in Fig. 6, which shows the relation between the frequency, the position and area of the closed curve of the node, respectively.

(*i*) (*ii*) (*iii*) (*iv*)

FIG. 6. The relation between the frequency, the position and area of the closed curve of the node:

	(*i*)	(*ii*)	(*iii*)	(*iv*)
f	6.33	6.50	6.60	6.80
h/l	0.785	0.786	0.787	0.788

f is frequency, h is height of the node, l is length of the beam (750 mm.).

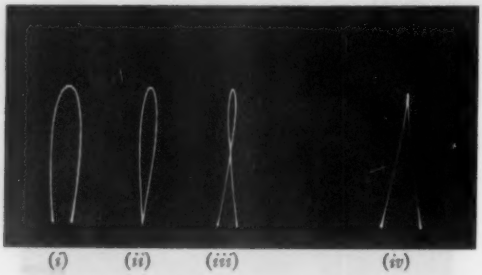
Similar phenomena about the damping of the area of the node may also be observed on the low frequency side of the case of Fig. 6(*i*), showing that the smaller the frequency, the smaller the area of the closed curve of the node.

FIG. 7. The three forms of the node.

From the above explanations, we arrive naturally at the conclusion that the point and straight line due both to the behavior of the node are no other than the mere special case of a straight line and of a closed curve, respectively. Thus, the three forms of the node mentioned above —a closed curve, a straight line and a point—are shown in Fig. 7.

MECHANISMS OF THE BEHAVIORS OF THE NODE AND THE POINT CLOSE TO IT

Continuing these studies in the position and movement in one period of the first node and the point close to it by marking them only with white spots, the authors obtained the several photographs shown in Fig. 8, of single curve of the path of the point above mentioned. Figures

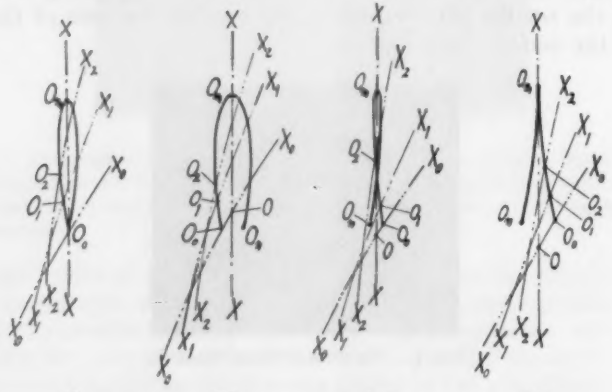


(*i*) (*ii*) (*iii*) (*iv*)

FIG. 8. (*i*), (*ii*) and (*iii*) show the path of a point just below the node, the node itself and a point just above it, respectively. (*iv*) represents the path of a point situated just above the point of (*iii*).

8(*i*, *ii* and *iii*) show the paths of a point just below the node, the node itself and a point just above the node. The path shown in Fig. 8(*iv*) represents that of a point situated just above the point shown in Fig. 8(*iii*).

The mechanism of the behavior of the node and its neighboring points can be explained qualitatively with the help of rough sketches.[6]



(*i*) (*ii*) (*iii*) (*iv*)

FIG. 9. Explanation of the mechanism of the movement of the node and its neighboring points (see Fig. 8).

[6] The paths of the node and its neighboring point such as, for example, the straight line, the curve having the knotted point in it, etc., are not explained analytically by using the usual theory of thin beams.

In Fig. 9(*i*), XX is the perpendicular at the position of zero amplitude of the beam, O_0 the lowest point of the node on the line. Let $X_0O_0X_0$ be the form of the beam near the first node when the clamped end is on the dead point of the stroke; the form of the beam near the node above mentioned at the next moment becomes $X_1O_1X_1$, $X_2O_2X_2$, $\cdots$. After $\frac{1}{4}$ period, it becomes XO_mO_0X on the perpendicular. Namely, the locus of the node in this case becomes $O_0O_1O_2 \cdots O_m$. Since the behaviors of the point near the node which are shown in Fig. 8(*i*, *iii* and *iv*) may similarly be explained using Fig. 9(*ii*, *iii* and *iv*), no further explanation is needed.

CONCLUDING REMARKS

The object of this paper is to present the photographs of the movements of the node and its neighboring points in an elastic beam at the time of stationary vibration. Phenomena which are common and natural enough to be lost sight of and left unnoticed are found everywhere. It seems to us that the movements of the first node and its neighboring points mentioned here may be a phenomenon akin to those mentioned above.

Rockets Probe Upper Atmosphere for IGY.—Five successful firings of Aerobee and Nike-Cajun high-altitude research rockets in the three-day period from January 25–27 were reported by the IGY Committee of the National Academy of Sciences. The firings are part of the IGY program of upper atmosphere studies and were conducted at a special rocket launching site built for the IGY at Fort Churchill, Canada.

The firings in this series were rated as good or excellent. The second one, on January 26, carried a 243-lb. payload 130 miles directly into an auroral display. It is expected that the data gathered by the instruments carried in this rocket will give scientists much additional information on the nature and formation of auroras and their relation to other events in the high atmosphere.

Since the beginning of the IGY, 13 of the 14 Aerobees fired at Churchill have been successful, and out of a total of 23 rockets fired at Churchill, 17 have been successful. The current series consisted of three Aerobees and two Nike-Cajuns.

The rockets carried their instrument pay-loads to heights ranging from 58 miles to 130 miles in experiments designed to yield information on the atmospheric structure above Fort Churchill. Air pressure, temperature, and density were measured at very high altitudes. The experiments for these rockets were designed by the University of Michigan and the Naval Research Laboratory.

The Fort Churchill rocket launching site was built by the Army Corps of Engineers and is managed by the Army Ordnance Corps with the cooperation of the Canadian IGY Committee and Canadian defense establishment. The site is particularly valuable for IGY studies because it lies well within the zone of maximum auroral intensity.

Other sites where the IGY is conducting upper atmosphere research by means of rockets include White Sands, New Mexico; Point Mugu, California; Guam; and ships in both the Arctic and Antarctic. Other rockets used by IGY scientists are the Aerobee-Hi (a version of the Aerobee used for altitudes up to 200 miles), the Nike-Asp, the Nike-Deacon, and Rockoon.

Radar Calculator.—A radar self-screening range calculator which provides a jamming figure of merit for radar system engineers has been designed by the Electronics Research Laboratory of Avco Research and Advanced Development Division, of Lawrence, Mass.

The hand calculator with its simplified scale settings eliminates slide rule calculations formerly required to determine the distance at which signal energy received at the radar is large enough to show through the jamming noise.

In addition to its applications in radar jamming and electronic counter measures, the Avco calculator has another advantage. The radar designer, having determined a minimum acceptable figure of merit, can also use the device to alter design parameters without recomputing his entire problem.

Original designers of the radar range calculator were William G. Madison, Assistant Engineer at Avco's Electronics Research Laboratory, and Alvin Mandell, now affiliated with Boeing Airplane Company.

Copies of the Avco calculator may be obtained by radar engineers through request on company letterhead to James McHugh, Director of Publications, Electronics Research Laboratory, Avco Research and Advanced Development Division, 750 Commonwealth Avenue, Boston, Mass.

NORMAL PERFORATION OF A THIN PLATE BY TRUNCATED PROJECTILES *

BY

BURTON PAUL [1] AND MELVIN ZAID [2]

ABSTRACT

The general solution to the problem of perforation of thin plates by high speed projectiles is extended to truncated conical and ogival projectiles. The velocity-distance relationships are obtained for these general cases, and the forces are evaluated for pointed ogival projectiles. Comparison of theory and experiment is made and the agreement is satisfactory.

1. NOMENCLATURE

a_1 = radius of projectile méplat
a_2 = radius of projectile body
F = force acting on projectile**
h = instantaneous thickness of target plate***
I = inertia effect on projectile momentum
K = experimental coefficient
L = line loading on projectile**
l_1 = distance from cone or ogive apex to méplat
l_2 = distance from cone or ogive apex to projectile body
$M(x)$ = effective mass of target plate
M_r = radial momentum of one petal
m = projectile mass
n = number of petals
R = radius of ogive
r = instantaneous radius to generic particle***
S = strength effect on projectile momentum
t = time
v = instantaneous projectile velocity
v_s = striking velocity of projectile
Δv = projectile velocity drop
x = penetration distance in axial direction
α = semi-vertex angle of projectile cone
β = angle of inclination of resultant line loading
θ = angle corresponding to x for ogival projectiles
ξ = instantaneous axial displacement of generic particle
ρ = mass density of target plate
τ_{max} = shear stress

* The original results in this paper were obtained in the course of research sponsored by Pitman Dunn Laboratories, Frankford Arsenal, under Contract No. DAI-36-038-507-ORD-(P)-339 and published in progress reports 1 to 8 (Project HEAP) by Bulova Research and Development Laboratories, Inc., Woodside, N. Y.

[1] Research Associate, Polytechnic Institute of Brooklyn, Brooklyn, N. Y.

[2] Technical Consultant, Plainview, L. I., N. Y.

** Subscripts "A," "r," refer to axial, radial direction.

*** Subscripts "o," "lim," "min," refer to original, maximum, minimum values.

2. INTRODUCTION

A general method for the analysis of the thin plate terminal ballistic problem has been presented in previous papers (**1**, **2**, **3**).[2] The basic approach to the problem adopted therein consists of an evaluation of the momentum imparted to the target material. This momentum, which has been shown to be equal to the momentum lost by the projectile, can be approximated by assuming a suitable deformation pattern for the target plate. This leads to great simplification when the material strength of the target is considered negligible in comparison to the extreme inertia forces produced at high speeds. It has been verified experimentally (**3**) that such an assumption leads to realistic results for the case of projectiles with conical noses. The present paper

FIG. 1. Typical target plate, showing petals rolled back after test.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

extends the analysis to truncated ogival and conical projectiles. Such configurations are the most commonly used in practice.

Experimental correlation is obtained using data from existing literature.

3. ANALYSIS

a. Slug Punching

For sufficiently high velocities, the main action of the projectile méplat[4] is to shear a disc from the plate equal in diameter to the méplat itself. Figure 1 shows the smooth hole of the same diameter as the méplat in a typical perforated target plate. The petals of this plate have been rolled back, showing that no large axial or circumferential stretching has occurred. The circumferential markings are from a collapsible windshield which was on the projectile. The momentum imparted to the slug (spall disc) is of importance in considering the total momentum of the target material. Therefore, the velocity of the slug must be determined. This can be written as

$$v_{slug} = K(v_s, h_o)v_s \tag{1}$$

where K lies between 1 and 2. The upper limit is obtained by assuming the slug is perfectly elastic and the projectile is a nondeforming solid of infinite mass, and the lower limit by considering that the slug stays with the méplat.

It is expected that as the plate thickness increases or as v_s decreases, the strength effects will become important and the value of K will be closer to its lower limit of 1. The correct value of K can be determined experimentally by firing flat nosed cylindrical projectiles against various targets and photographing the relative positions of projectile and slug at short time intervals. A typical photograph from a series of tests[5] using aluminum plates and projectiles is shown in Fig. 2. On the basis of these and similar type tests, values of K have been calculated, as shown in the accompanying table.

Target Thickness, in.	Striking Velocity, ft./sec.	K
0.04	2510	1.33
0.125	2590	1.23
0.04	900	1.16
0.125	900	1.12

These are typical results, and can be used as a guide in the selection of K.

The inertia and strength effects can be compared in the following

[4] "Méplat" is a ballistic term meaning "a flat surface on the nose tip."

[5] This experimental work was performed by Mr. William Davis of BRL, Aberdeen Proving Ground, Md. (1955) and recorded in Firing Records S-46144 and S-46176 by Development and Proof Services, APG.

manner. The projectile momentum change due to the inertia effect is

$$I = \pi a_1^2 h_o \rho K v_o. \tag{2}$$

The momentum change due to strength effect is approximately

$$S = 2\pi a_1 h \tau_{max} t \tag{3}$$

where t is the time during which τ_{max} acts and h is the effective thickness on which it acts. For the simple slug punching process envisioned,

$$t \leq \frac{h_o}{v_o} \tag{4}$$

$$h \leq h_o \tag{5}$$

so that

$$S \leq \tau_{max} \frac{2\pi a_1 h_o^2}{v_o}. \tag{6}$$

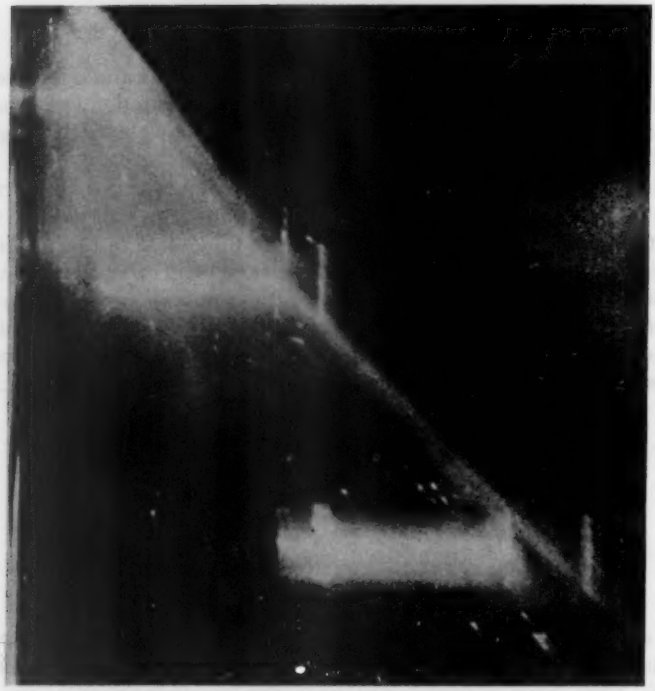


FIG. 2. Velocity loss during perforation of dural plate. The series of micro-flash pictures, taken with open shutter fastax camera shows successive views (from top to bottom) of projectile and spall disc after perforation. Projectile is 30-mm. aluminum cylinder 3.84 in. in length, with a velocity of approximately 2000 ft. per second. Plate thickness, 0.064 in.

Thus

$$\frac{S}{I} \leq \frac{2}{K}\left(\frac{\tau_{max}}{v_s^2\rho}\right)\frac{h_o}{a_1}. \tag{7}$$

For sufficiently high striking velocities and thin plates this strength effect is negligible. As an example, for an aluminum target with

$$\frac{h_o}{2a_1} \approx 0.1, \qquad v_s = 2000 \times 12 \text{ in./sec.}, \qquad \tau_{max} \approx 50 \times 10^3 \text{ psi.},$$

$$S \leq 13\% \, I$$

Since the momentum loss associated with the spall disc is only a part of the entire momentum loss for many projectile configurations, this order of magnitude of strength effect can be considered negligible. However, since the ratio of S/I varies as $1/v_s^2$, it is apparent that at sufficiently low v_s, the strength effect becomes important. This is confirmed by experimental evidence.

b. Perforation by Truncated Conical Projectile

It will be assumed that the slug is punched without deformation of the surrounding plate material. The cone then forms petals as described previously (3). From reference 3 (Eqs. 5 and 7), and consideration of slug momentum, the projectile velocity drop is given by

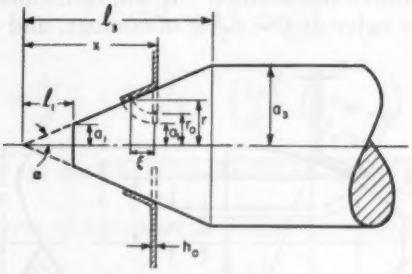


FIG. 3.

$$\Delta v = \frac{Km_{slug}v_s + M(x)v}{m} \tag{8}$$

where

$$M(x) = 2\pi\rho h_o \int_{r_{o_{min}}}^{r_{Lim}} r_o \frac{\partial \xi}{\partial x}\, dr_o. \tag{9}$$

From Fig. 3

$$\xi = (x \tan \alpha - r_o) \cos \alpha. \tag{10}$$

Considering $\Delta v \ll v_s$, Eq. 8 becomes

$$\Delta v(x) = \frac{\pi \rho h_o}{m} v_s \tan^2 \alpha [K l_1^2 + (x^2 - l_1^2) \sin \alpha]. \tag{11}$$

As explained in reference 3, perforation is complete when $x = l_3$.

The total velocity drop is therefore:

$$\Delta v_{total} = \frac{\pi \rho h_o}{m} v_s [K a_1^2 + (a_3^2 - a_1^2) \sin \alpha]. \tag{12}$$

For the assumed model, once the slug is punched, corresponding elemental particles in the truncated and full conical projectile have identical motion. Since these particles experience no momentum change once they become part of a petal, it follows that the entire axial or radial force results from the velocity change of the particle at the intersection of the target and projectile. Thus, excluding the slug punching effect, the axial and radial line loadings, etc. are identical to those already obtained for the full conical projectile (3).

c. Velocity Drop of Truncated Ogival Projectile

The previously developed general theory (1, 2, 3) will be used to obtain all the results of this section. It will be assumed that the petals are tangent to the ogive at the circle of contact, and that they exhibit

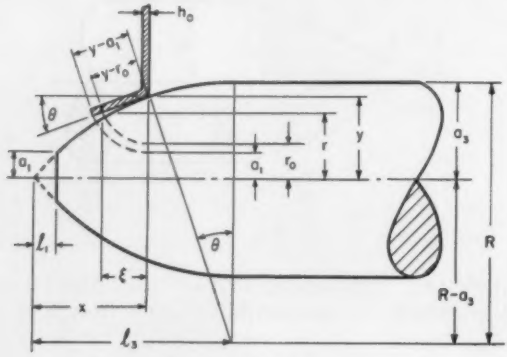


FIG. 4.

no stretching. This is illustrated in Fig. 4. The velocity drop is then obtained from Eqs. 8 and 9.

The forward movement of a generic particle is

$$\xi = (y - r_o) \cos \theta. \tag{13}$$

From Fig. 4

$$x = l_3 - R \sin \theta \tag{14}$$

$$\frac{d\theta}{dx} = -\frac{1}{R \cos \theta} \tag{15}$$

$$\frac{dy}{dx} = \tan \theta = \frac{l_3 - x}{y + (R - a_3)}. \tag{16}$$

Taking the x derivative of Eq. 13 and using Eqs. 9, 15 and 16 in 8 with the consideration that $\Delta v \ll v_s$, there results

$$\Delta v(x) = \frac{2\pi\rho h_o R^2}{m} v_s \left[\frac{1}{2} K \left(\frac{a_1}{R}\right)^2 + \frac{1}{6}\frac{l_3}{R}\left(\frac{y}{R} - \frac{a_1}{R}\right)\left(1 - \frac{x}{l_3}\right)\right] \times \left\{3\left(\frac{y}{R} + \frac{a_1}{R}\right) + \frac{\left(\frac{y}{R} - \frac{a_1}{R}\right)\left(\frac{y}{R} + 2\frac{a_1}{R}\right)}{\left(\frac{y}{R} + 1 - \frac{a_3}{R}\right)}\right\}. \tag{17}$$

From Fig. 4

$$\frac{l_3}{R} = \sqrt{1 - \left(1 - \frac{a_3}{R}\right)^2} \tag{18}$$

$$\frac{y}{R} = \frac{a_3}{R} - 1 + \sqrt{1 - \left(\frac{l_3}{R}\right)^2 \left(1 - \frac{x}{l_3}\right)^2}. \tag{19}$$

Equations 17 to 19 are used to compute $\Delta v(x)$ for any projectile defined by a_1, a_3, and R.

For the special case of a pointed projectile ($a_1 = 0$), Eq. 17 reduces to

$$\Delta v(x) = \frac{2\pi\rho h_o R^2}{m} v_s^2 \frac{l_3}{R}\left(\frac{y}{R}\right)^2 \left(1 - \frac{x}{l_3}\right)\left\{\frac{1}{2} + \frac{1}{6}\frac{y/R}{\left(\frac{y}{R} + 1 - \frac{a_3}{R}\right)}\right\}. \tag{20}$$

Since there can be no tensile forces between the projectile and petal, contact is maintained only as long as the petal momentum is monotonically increasing. The petals separate from the projectile when

$$\frac{\partial}{\partial x}(\text{petal momentum}) = 0$$

$$\frac{\partial^2}{\partial x^2}(\text{petal momentum}) < 0. \tag{21}$$

For the profile and basic model under consideration, contact will not be re-established.

The petal momentum is zero at $x = l_1, l_3$ and is always greater than or equal to zero. Therefore by Rolle's theorem the x derivative of petal momentum (and thus of Δv) must vanish before complete perforation, and there will always be a separation point (x^*) within the limits

$$l_1 \leq x^* \leq l_3. \tag{22}$$

Then

$$\Delta v_{total} = \Delta v(x^*). \tag{23}$$

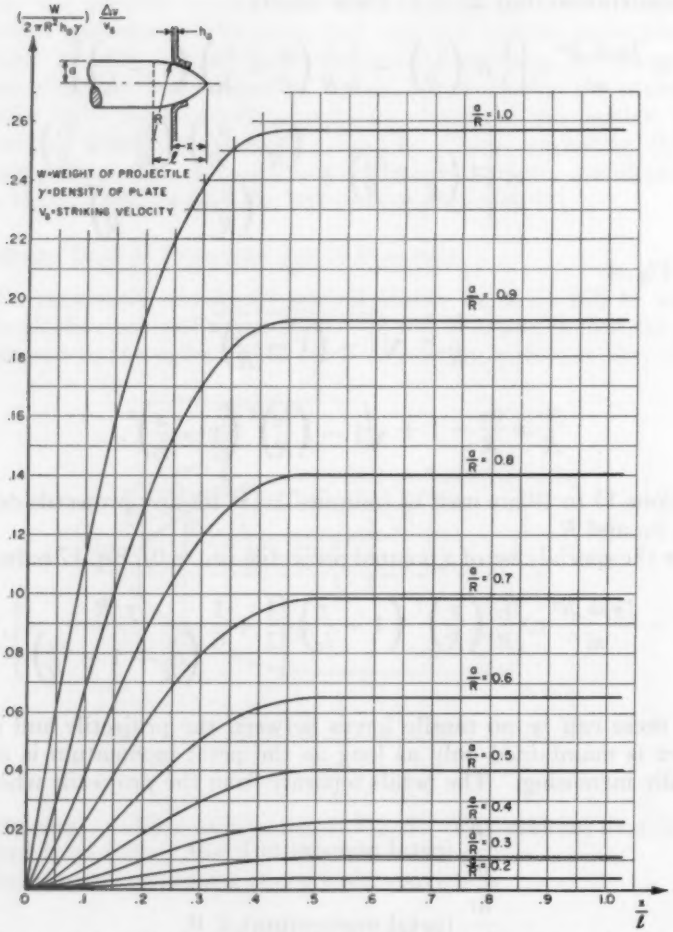


FIG. 5. Velocity drop (Δv) of ogival projectile during penetration of thin plate.

The graphs of Δv *versus* x and Δv_{total} *versus* a_3/R for the pointed ogival projectile are presented in dimensionless form in Figs. 5 and 6, re-

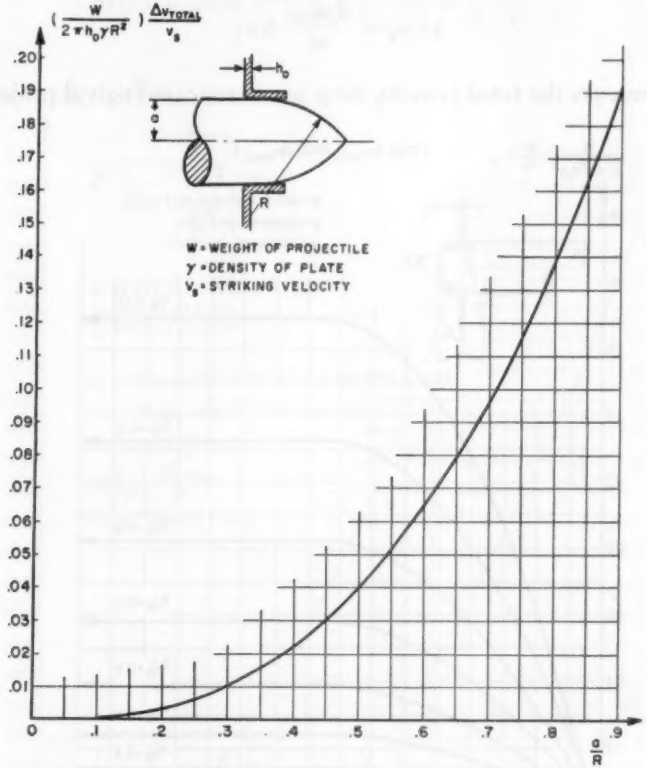


FIG. 6. Total velocity drop (Δv_{total}) for ogival projectile.

spectively. For $a_3/R > 0.12$, Fig. 6 can be represented by

$$\Delta v_{total} = \left(\frac{2\pi\zeta h_o R^2 v_s}{m}\right) 0.249 \left(\frac{a_3}{R}\right)^{2.60} \tag{24}$$

within $\pm$ 3 per cent accuracy.

One should note that as $a_3/R \to 1$, the ogive becomes a hemisphere. For this case, it is expected that slug punching with $K = 1$ will occur. Thus care must be exercised in extending the figures to the limit case of $a_3/R = 1$.

A typical dimensionless plot[6] of $\Delta v(x)_{petals}$ for the truncated ogival

[6] See HEAP Progress Report No. 7, Bulova Research and Development Laboratories, Inc., 15 April 1956, Contract DAI-36-038-507-ORD-(P)-339 for more detailed results.

projectile is presented in Fig. 7, where it is noted that $x' = x - l_1$, $l = l_3 - l_1$. The slug Δv must be added separately and is

$$\Delta v_{slug} = \frac{\pi \rho h_o v_s}{m} K a_1^2. \tag{25}$$

Fig. 8 presents the total velocity drop for a truncated ogival projectile.[6]

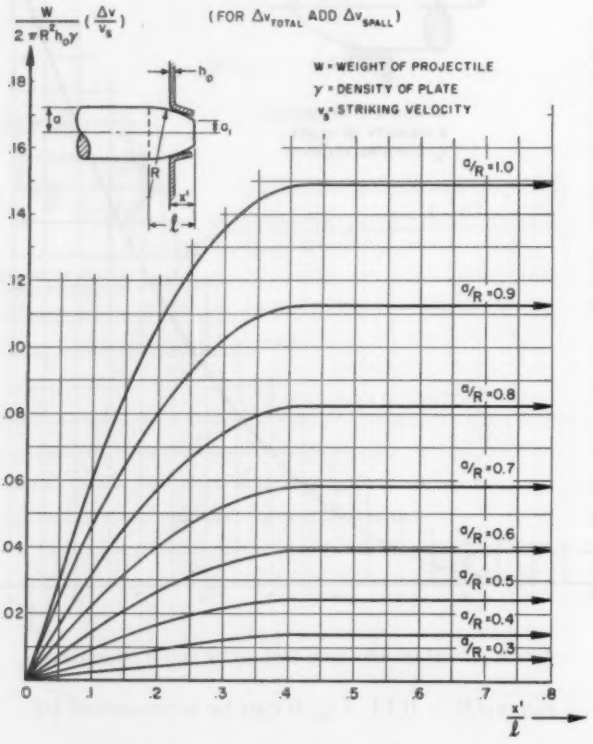


FIG. 7. Petal formation. Velocity drop (Δv) of ogival projectile with 0.5 caliber méplat ($a_1/a = 0.5$) during normal penetration of thin plate.

d. Forces Acting on Pointed Ogival Projectile ($m_{slug} = 0$)

(i) Axial Force

It has been shown (3) that for $\Delta v \ll v_s$

$$F_A = v_s^2 \frac{dM(x)}{dx}. \tag{26}$$

From Eqs. 8, 14, 16 and 20, (taking the correct value of v in Eq. 20),

$$M(x) = 2\pi\rho h_o R^2 \left(\frac{y}{R}\right)^2 \left[\frac{\sin\theta}{2} + \frac{y}{R}\frac{\tan\theta}{6}\right]. \tag{27}$$

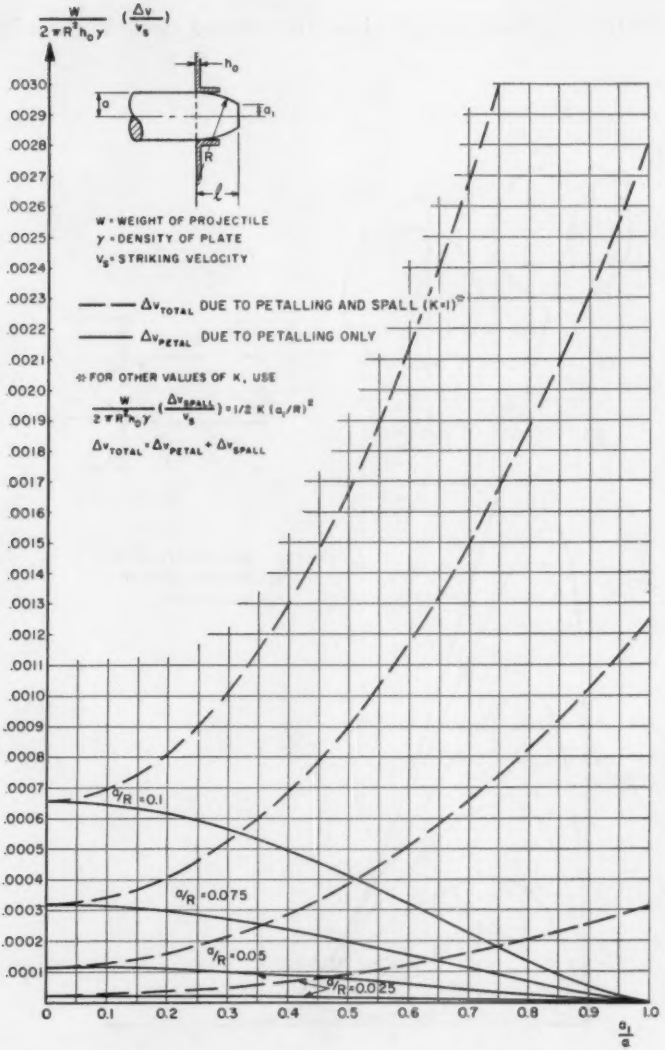


FIG. 8. Total velocity drop (Δv) for ogival projectiles with méplats.

By combining Eqs. 15, 16, 26 and 27 and letting $\Delta v \ll v_s$, the axial force is obtained as

$$F_A = 2\pi\rho h_o v_s^2 y \left\{ \tan\theta \sin\theta + \frac{1}{2}\frac{y}{R}(\tan^2\theta - 1) - \left(\frac{y}{R}\right)^2 \frac{1}{6\cos^2\theta} \right\}. \quad (28)$$

The axial force per unit length along the contact circle is given by

$$L_A = \frac{F_A}{2\pi y}. \quad (29)$$

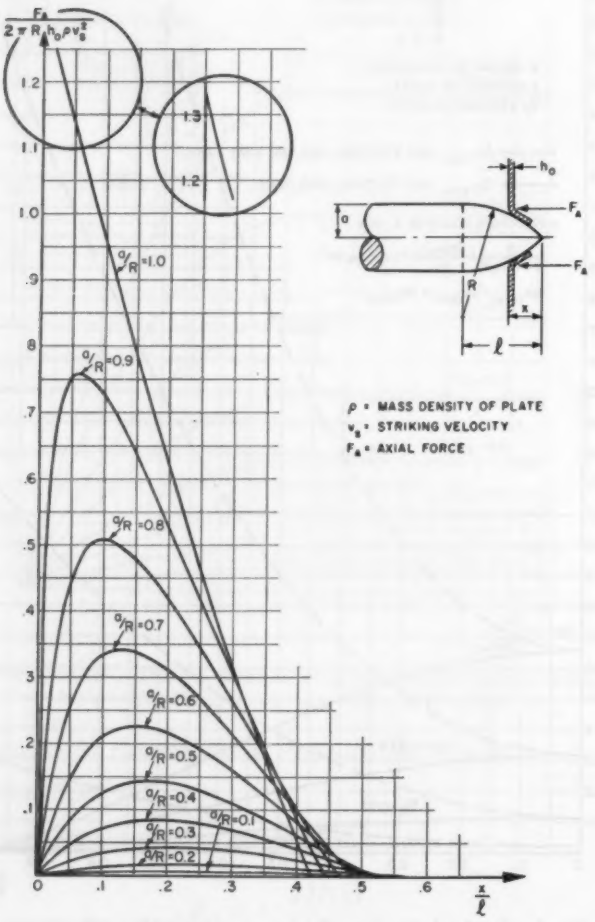


FIG. 9. Axial force (F_A) on ogival projectile.

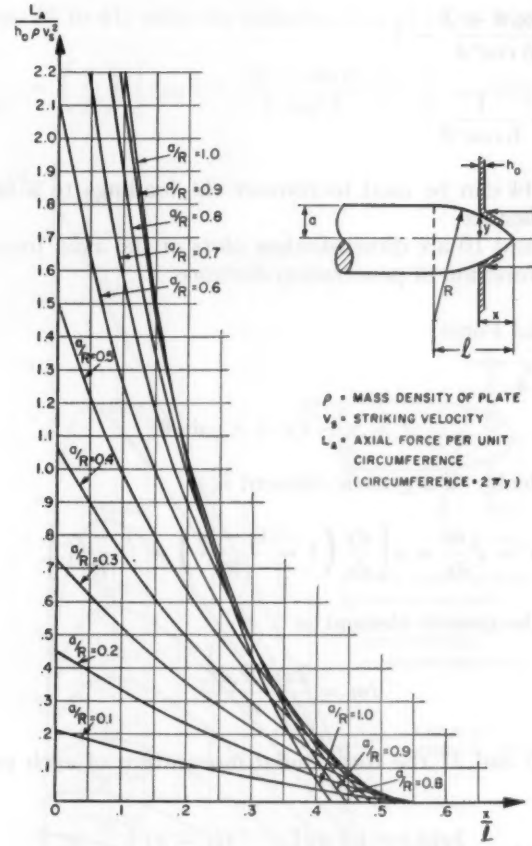


FIG. 10. Axial line loading (L_A) on ogival projectile.

By using the relationship that

$$\frac{y}{R} = (\cos\theta - 1) + \frac{a_3}{R} \tag{30}$$

and substituting Eq. 28 into 29, the axial "line loading" is obtained as

$$L_A = \rho h_c v_s^2 \left[c_0 + c_1 \frac{a_3}{R} + c_2 \left(\frac{a_3}{R} \right)^2 \right] \tag{31}$$

where

$$c_0 = \tan\theta \sin\theta - (1 - \cos\theta) \left[\frac{\tan^2\theta}{2} + \frac{1 - \cos\theta}{6\cos^2\theta} - \frac{1}{2} \right] \tag{32}$$

$$c_1 = \frac{\cos\theta + 2}{6\cos^3\theta} - 1 \tag{33}$$

$$c_2 = -\frac{1}{6\cos^3\theta}. \tag{34}$$

Equation 14 can be used to convert the loadings to a function of penetration distance.

Figures 9 and 10 are dimensionless plots of the axial force and line loadings as a function of penetration distance.

(*ii*) Radial Force

From Fig. 4:

$$r = y - (y - r_o)\sin\theta. \tag{35}$$

The radial velocity of a generic element is

$$\dot{r} = v\frac{\partial r}{\partial x} = v\left[\frac{dy}{dx}\left(1 - \frac{l_3 - x}{R}\right) + \frac{y - r_o}{R}\right]. \tag{36}$$

The mass of the generic element is

$$dm = \frac{2\pi\rho h_o r_o dr_o}{n}. \tag{37}$$

Using Eqs. 36 and 37 the total radial momentum of each petal is obtained as

$$M_r = \frac{2\pi\rho h_o v y^2}{n}\left\{\frac{1}{2}\frac{dy}{dx}\left[1 - \frac{(l_3 - x)}{R}\right] + \frac{y}{6R}\right\}. \tag{38}$$

The radial force on each petal is given by

$$F_r = v\frac{dM_r}{dx}. \tag{39}$$

Now writing Eq. 38 as

$$M_r = v\,f(x) \tag{40}$$

it can be shown[6] that for the ogival projectile, considering $\Delta v \ll v_s$,

$$\frac{dM_r}{dx} \approx v_s\frac{df(x)}{dx}. \tag{41}$$

From Eqs. 38 to 41, with the substitutions of 14 to 16, there results

$$\frac{nF_r}{2\pi\rho h_o R v_s^2} = \left(\frac{y}{R}\right)^2 \left[\tan\theta - \frac{(1-\sin\theta)}{2\cos^3\theta}\right] + \frac{y}{R}\tan^2\theta(1-\sin\theta) \quad (42)$$

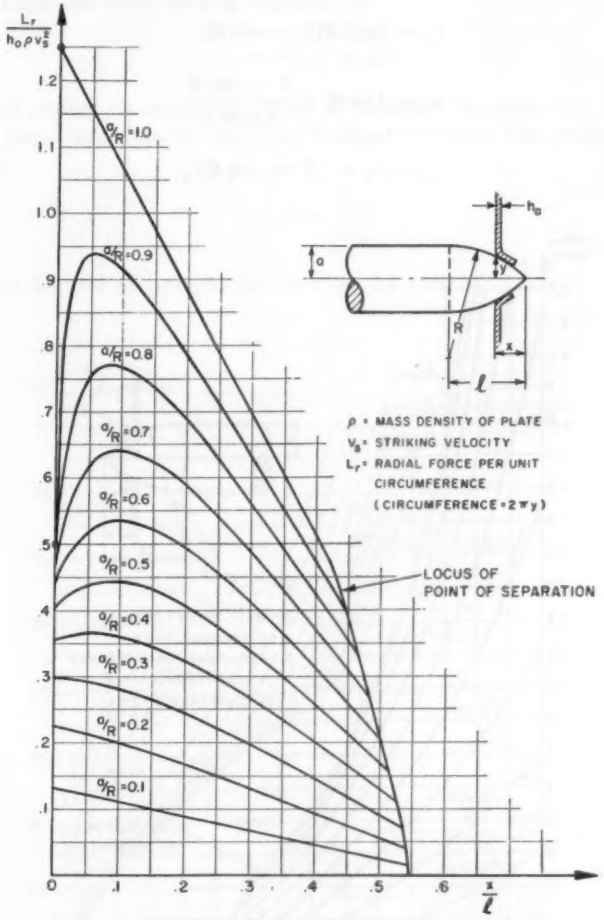


FIG. 11. Radial line loading (L_r) on ogival projectile.

The radial "line loading" is given by

$$L_r = \frac{nF_r}{2\pi y} \quad (43)$$

so that, using Eq. 30,

$$\frac{L_r}{\rho h_o v_s^2} = c_5 + c_4 \frac{a_3}{R} \tag{44}$$

where

$$c_3 = \tan^2\theta(1 - \sin\theta) \tag{45}$$

$$c_4 = \tan\theta - \frac{1 - \sin\theta}{2\cos^3\theta} \tag{46}$$

$$c_5 = c_3 - (1 - \cos\theta)c_4. \tag{47}$$

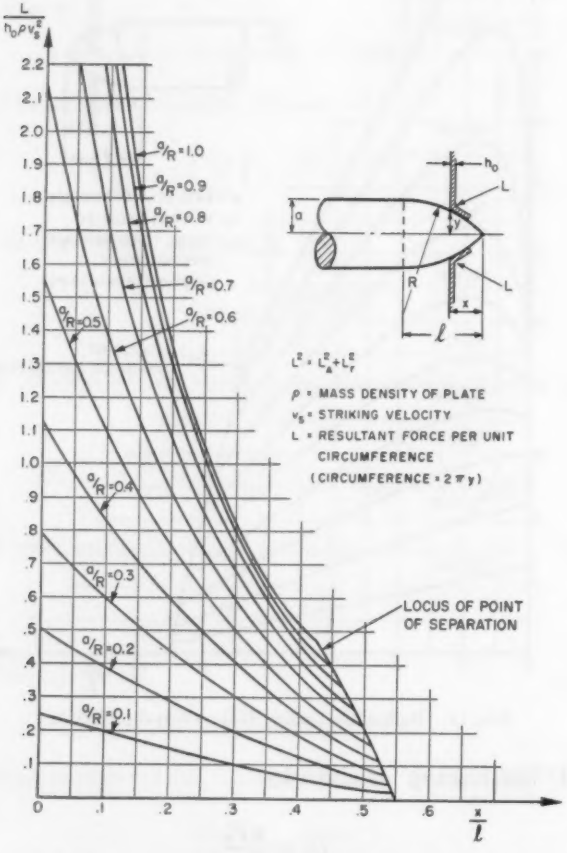


FIG. 12. Resultant line loading (L) on ogival projectile.

Figure 11 is a dimensionless plot of the radial line loading as a function of penetration distance.

(iii) Resultant Force and Direction

The resultant line loading is given by

$$L = \sqrt{L_A^2 + L_r^2}. \tag{48}$$

This load passes through the circle of contact between the projectile and the petal and has an angle of inclination from the projectile axis given by

$$\tan \beta = \frac{L_r}{L_A}. \tag{49}$$

Equations 48 and 49 are plotted in Figs. 12 and 13.

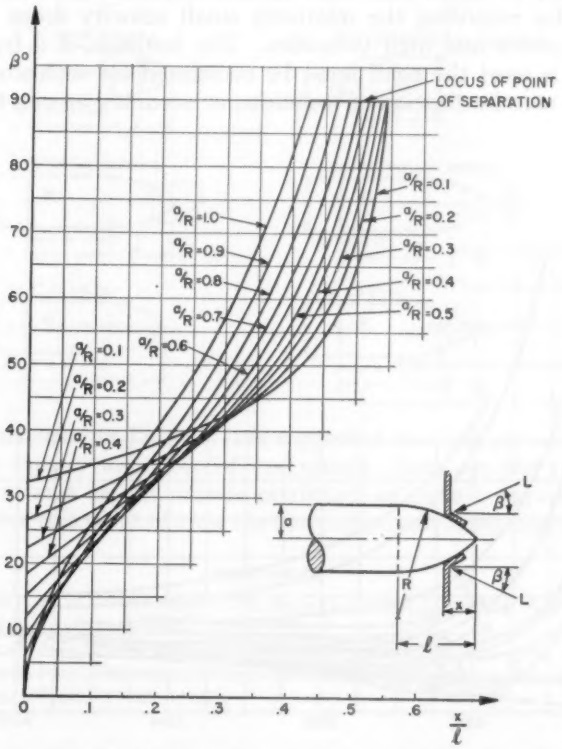


FIG. 13. Angle of incidence (β) of resultant line loading on ogival projectile.

4. EXPERIMENTAL CORRELATION

Values of $\frac{(\Delta v) \text{ actual}}{(\Delta v/v_s) \text{ theory}}$ are plotted against v_s in Fig. 14 for numerous projectiles with various profiles. The experimental data were taken from the various sources indicated in Table I.

Since the material strength effects will predominate over the inertia effects at low velocities, the analysis presented in this paper is not expected to be accurate in that region. However, the accuracy should become increasingly better for higher striking velocities, and similarly for thinner target plates. The experimental results show both these trends, and the velocity drop approaches the predicted behavior asymptotically as striking velocity increases and target thickness decreases.

It is quite likely that a static analysis accounting for material strength effects could supplement the present theory in the low velocity, thick target range.

It should be noted that very careful and accurate techniques are necessary for recording the relatively small velocity drops associated with thin plates and high velocities. For instance, if a transmission pendulum is used the spall must be accounted for separately. Since there is no standardization of technique or accuracy among the various

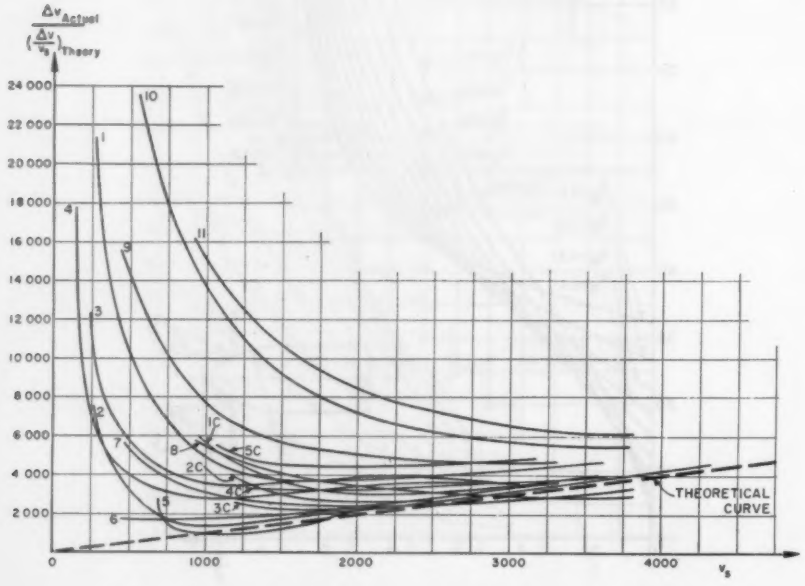
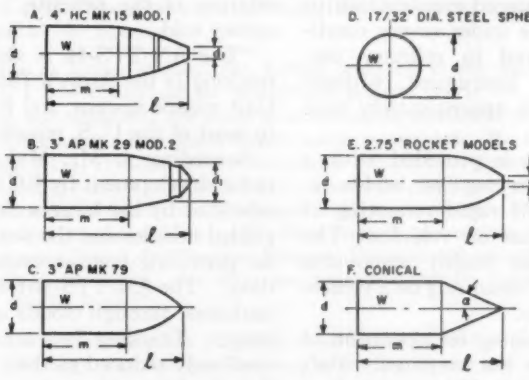


FIG. 14. Correlation between experiments and theory (see Table I).

TABLE I—*Experiments Recorded in Fig. 14.*

Curve No.	Proj. Type	Plate Mat'l.	Plate Thick (in.)	ℓ (in.)	m (in.)	α (°)	d (in.)	d_1 (in.)	W (#)	Reference
1	A	Hot Rolled Steel	.126	6.571	3.219	—	1.46	0.19	1.610	NRL 0-2891 Beij
2	B		.126	3.839	2.177	—	1.46	1.24	1.519	
3	C		.126	4.503	2.183	—	1.46	0	1.741	
4	C		.076							
5	D	24 ST AL	.032	—	—	—	17/32	—	0.0236	N.M.School of Mines TR-459 Rinehart, Zandstra
6	E	2-S AL	.020	9.67	5.44	—	1.46	0.172	1.35	NRL Memorandum Rept. 150 Clark, Hassel, Krafft
7	E		.050							
8	E		.090							
9	E		.020	3.31	1.86	—	0.50	0.059	0.05	
10	E		.050							
11	E		.090							
1C	A	Cold Rolled Steel	1/32	2-3/8	Analysed as Cone (F)					NOLM 5977 Melzelaar
					—	11	1/2	—	—	
2C	F	24 ST AL	.051	3-1/4	—	10	0.79	—	0.242	Frankford Arsenal Pitman-Dunn Labs. J. Kymer (1956)
3C	F		.051	2-1/2	—	20	0.79	—	0.242	
4C	F		.072							
5C	F		.091							



experimenters cited in Table I, it is suggested that the curves shown in Fig. 14 are merely an order of magnitude check on the theoretical velocity drop and on the trends predicted as certain parameters are varied. From this point of view the correlation exhibited is satisfactory.

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(2) M. ZAID AND B. PAUL, "Armor Penetration Survey," *Ordnance*, January, 1956, pp. 609–611.

(3) M. ZAID AND B. PAUL, "Mechanics of High Speed Projectile Perforation," JOUR. FRANKLIN INST., Vol. 264, pp. 117–126 (1957).

Improved Sun Compass.—A sun compass with improved accuracy, versatility and operational range has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va.

Now military standard, the new universal sun compass provides a means for land navigation at all north or south latitudes, and can be used with many navigational stars. A clock mechanism provides for the mechanical tracking of the sun or stars. Its predecessor was limited in its daytime operation from the equator to 45 deg. north or south latitudes and it could be used only with the North Star.

Although simple in design, light in weight, and compact, the improved compass is of rugged construction to provide long life under severe conditions encountered in military use. Weight of the instrument, without carrying case, is approximately nine pounds.

The compass is provided with a weatherproof carrying case, and is designed to permit rapid mounting of case and compass on vehicles. The compass is also readily removable from its case for mounting on a vehicle or for other uses.

Specialist training for operation of the compass is not required. Unit training in advanced map reading, supplemented by approximately two hours instruction, or previous experience, in star identification, and one hour in actual operation of the sun compass will suffice for the average soldier.

Radars Used to Track Satellite Launching.—When the Army's Explorer rose into its history-making orbit, its flight was followed by two of the most accurate tracking radars in the world, it was disclosed by Arthur L. Malcarney, Executive Vice President, Defense Electronic Products, of the Radio Corporation of America.

A number of these instruments, Mr. Malcarney said, are currently being produced in quantity by the Radio Corporation of America, for installation on Army, Navy and Air Force missile ranges throughout the country. They are the first specifically designed for guided-missile range instrumentation, and are the result of a ten-year program of Research and Development carried out by RCA in conjunction with the Applied Physics Laboratory of Johns Hopkins University, the Navy Bureau of Ordnance and the Navy Bureau of Aeronautics.

"The radars, located on the Florida coast and on Grand Bahama Island, 165 miles away, provided information vitally necessary for the successful orbiting of the satellite," Mr. Malcarney said.

"The AN/FPS-16 is also used for tracking in the Army's Talos Defense Unit missile system, and is adaptable to most of the U. S. missile systems."

According to Mr. Malcarney, the radar development by RCA, "was accelerated by the large-scale testing of guided missiles and the resultant need for providing fast, accurate, tracking data. The AN/FPS-16 tracks in total darkness, through clouds and at long range. Tracking data are almost immediately reduced to their final form. Previously, weeks were required to translate tracking data to intelligible form.

"Still another feature of this highly accurate radar is its ease of control with only a single operator needed for each unit. It is also reliable, functional under all weather conditions, flexible and highly standardized for use by all three services—thus eliminating duplication of effort.

"Along with the more glamorous missiles that soar majestically into the sky every day, this precision radar is playing a tremendously important supporting role in the drama of national defense."

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

March 19, 1958

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. W. Laurence LePage, President, called the meeting to order with approximately 220 members and guests in attendance, including members of the Rittenhouse Astronomical Society.

Mr. LePage stated the minutes of the Annual Meeting of January 15, 1958, were published in the February issue of the JOURNAL. There being no corrections or additions, the minutes were approved as published. Mr. LePage announced that the minutes of the Stated Meeting of February 19, 1958 will be published in the March issue of the JOURNAL and will be presented for approval at the next meeting.

There being no further business, Mr. LePage proceeded to the main activities of the evening, giving a brief background on "fusion" and peaceful uses of the reaction, the topic for the evening, and noting that this would be an interesting occasion in that the Rittenhouse Medal of the Rittenhouse Astronomical Society was to be awarded. Mr. LePage then introduced Mr. Emil Klein, President of the Rittenhouse Astronomical Society. Mr. Klein extended greetings and asked Dr. I. M. Levitt, a member of the Board of Governors of the Rittenhouse Astronomical Society and Director of The Franklin Institute Planetarium, to read the citation for the Rittenhouse Medal. Mr. Klein made the presentation of the Rittenhouse Medal to Dr. Lyman Spitzer, Jr., Director, Princeton University Observatory, who expressed deep appreciation for the honor. Mr. Klein then announced that Dr. Spitzer would speak on the subject of "Fusion Power."

Dr. Spitzer's talk, showing how the study of astronomy has led to the possibility of an infinite supply of energy, was presented in a very interesting manner and was extremely well received by the audience as evidenced by the many questions which Dr. Spitzer ably answered following his talk.

After a rising vote of thanks to Dr. Spitzer, Mr. LePage adjourned the meeting at 9:25 P.M.

WILLIAM F. JACKSON, JR.
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, February 12, 1958.)

MR. JOHN H. NEHER in the Chair.

The following report was presented for final action:

No. 3316: Flame Weeding of Cotton.

This report recommended the award of an Edward Longstreth Medal to Price C. McLemore, of Waugh, Alabama, "For his successful application of selective flame cultivation to reduce the life expectancy of noxious growths in a cotton field together with the large reduction in labor costs, and the fostering of more complete farm mechanization."

(Abstract of Proceedings of Stated Meeting held Wednesday, March 12, 1958.)

MR. JOSEPH GRAY JACKSON in the Chair.

The following report was presented for final action:

No. 3308: Work of Stephen P. Timoshenko.

This report recommended the award of an Elliott Cresson Medal to Stephen P. Timoshenko, of Palo Alto, California, "In consideration of his contribution to Applied Mechanics as an author and teacher, and for his original work in the Theory of Elasticity and Elastic Stability."

D. S. FAHRNEY,
Secretary to Committee

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Since 1824 The Franklin Institute, through wise and conservative management, has been partially self-supporting.

As an old institution, established in the public interest, The Franklin Institute has tried to earn most of its cost by diligent and productive work. Like many other institutions of a non-profit nature, The Institute has, in these years of an expanding economy and rising spiral of costs, found it increasingly difficult to perform the broad educational services of which the growing nation and territories are ever in need.

Despite an erroneous public impression gained, perhaps, by the excellence and wide range of services performed, that it is a wealthy, richly endowed organization, The Institute always needs funds.

There is a warm satisfaction in giving financial support to an organization that will use it constructively to help advance and improve our type of civilization.

Your gift or bequest, large or small, will be deeply appreciated. We shall be pleased to give any additional information regarding gifts or memorials.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

Please write to: The Secretary, The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania.

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THE FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

DESIGNING HYDROSTATIC BEARINGS FOR MINIMUM PUMP POWER CONSUMPTION [1]

BY

A. M. LOEB [2] AND H. C. RIPPEL [2]

A new technique has been devised by the staff of the Friction and Lubrication Section of the Laboratories for determining the optimum proportions of a hydrostatic bearing, from the standpoint of minimum pump power consumption.

Because the cost of operating a large or complex hydrostatic bearing system can be very high, especially when the bearings are in con-

FIG. 1.

[1] Excerpt from a paper entitled "Determination of Optimum Proportions for Hydrostatic Bearings through the Use of the Electric Analog Field Plotter," by Loeb and Rippel, which has been scheduled for presentation at the April, 1958 Annual Meeting of the American Society of Lubrication Engineers.

[2] Research Engineers, Friction and Lubrication Section, Mechanical Engineering Division.

tinuous operation, it is important for the designer to consider the effect of various bearing configurations on the amount of pump power that will be consumed in the bearing. Usually, he finds that the external size of the bearing is dictated by the space available; the main question then is: what size and shape should the bearing and bearing recess have? In other instances, pressure and flow limitations will be established by the pump capacity or other factors.

A technique based on the electric analog field plotter[3] has been useful in determining the optimum proportions of a bearing, which will

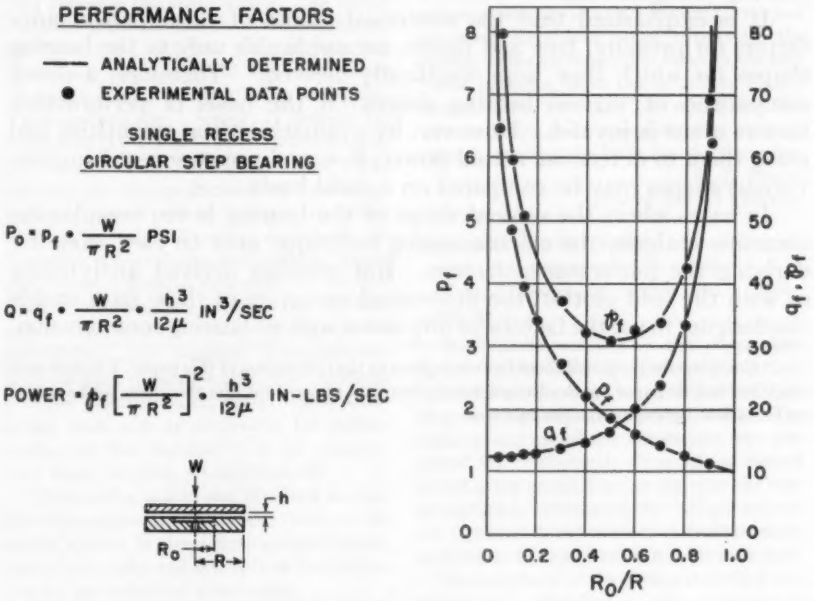


FIG. 2

cause it to operate at minimum pump power, or enable it to meet such other requirements as those noted above. With this method, for example, we have found that in design of hydrostatic bearings to support large rotating antennas of the steerable paraboloid type a considerable saving in power can result from proper selection of recess size.

This technique was used in determining the bearing proportions for the hydrostatic bearings being used to support the 140-ft. diameter radio telescope to be installed at Greenbank, West Virginia, by the Associated Universities. Figure 1 shows a model of the bearings (surfaces painted white) and support structure for this telescope.

[3] A. M. LOEB, "Electric Analog Study of Hydrostatic Bearings," JOUR. FRANKLIN INST., Vol. 263 p. 450 (1957).

The effect of recess size on the pressure, flow and power requirements of several hydrostatic bearing configurations have been defined in terms of dimensionless coefficients we term "performance factors." These factors have also been determined analytically for a simple circular hydrostatic step bearing. The results of this analysis were then compared to the electric analog solution. As shown in Fig. 2, this comparison shows excellent agreement. Also, these curves show that a particular recess size (in this case, when the radius of the recess is 0.53 the radius of the bearing) will give minimum pump power for this bearing configuration.

It is emphasized that the numerical values of these performance factors for pressure, flow and power, are applicable only to the bearing shapes for which they were specifically derived.[4] Therefore, a direct comparison of various bearing shapes on the basis of performance factors alone is invalid. However, by evaluating these quantities and using them to determine actual power, flow and pressure requirements, various shapes may be compared on a valid basis.

In cases where the desired shape of the bearing is too complex for accurate analysis, the electric analog technique may be used alone for deriving the performance factors. But whether derived analytically or with the field plotter, the dimensionless nature of these data enable the designer to use the factors for any size of a given bearing configuration.

[4] Complete derivation of these factors is given in the full version of this paper. The approach described in this paper applies to hydrostatic bearings of constant film thickness, and assumes no cross-flow between adjacent recesses.

BOOK REVIEWS

THERMODYNAMICS OF HEAT-POWER SYSTEMS, by F. W. Hutchinson. 490 pages, diagrams, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1957. Price, $8.50.

This is a textbook suitable for an introductory course in thermodynamics. The author chooses a welcome innovation and has divided his book into 7 sections. Each of these sections deals with a different step in the study of energy from its source to its utilization. These sections are: I Source; II Storage; III Liberation; IV Transition: V Transformation; VI Transportation, and VII Utilization. The section under storage discusses the three major forms of stored energy—atomic, molecular and mechanical. Liberation deals with the liberation of heat during combustion; Transition covers a detailed treatment of heat and work; Transformation involves the flow and non flow process, the concept of availability, reversibility and entropy. Transportation covers a study of gases, vapors, their mixtures and the thermodynamic cycle; finally Utilization deals with an analysis of the various cycles and their application to air compressors, steam turbines, refrigeration, etc.

As the author points out, this book emphasizes ideas rather than equations; and it is the author's intent to give a description of fundamental principles and concepts so that extension by the instructor is necessary.

In this philosophy, the reviewer agrees with the author. The book is clearly written and contains excellent sections on combustion, heat and work. Several illustrative problems are given in each chapter. The section dealing with thermodynamic cycles contains two 11 × 17 charts of pressure-enthalpy and entropy-temperature for the water steam system. These charts are well reproduced with red figures on a white background.

Several rather obvious errors have occurred in printing, for instance example 12-11 on p. 269 lists a value of 1180 BTU/lb on Fig. 12-8 whereas this value should be 1160. The differential sign is omitted in the mathematical statement of entropy.

Dalton's Law is stated as applying to all gas vapor mixtures, but this law applies strictly to the ideal gas only, large deviations being known to exist at high pressures.

This reviewer would liked to have seen more extensive space and comments devoted to units; a more fundamental derivation of the gas constants and at least a subscript attached to pound force and pound mass since the author makes no distinction between the two. These foregoing items can of course, be discussed by the instructor but would be otherwise confusing to anyone unfamiliar with the subject, reading the book for the first time.

E. J. NOLAN
General Electric Company

BOUNDARY LAYER EFFECTS IN AERODYNAMICS. v.p., illustrations, 8¼ × 10½ in. New York, Philosophical Library, 1957. Price, $12.00.

This volume consists of the papers presented at the NPL symposium "Boundary Layer Effects in Aerodynamics" held at Teddington, England, in the Spring of 1955. The opening address of the symposium was presented by Professor L. Howarth of Bristol covering the progress in the theory of the two-dimensional, incompressible, laminar boundary layer and its extension to three-dimensions and thence to supersonic and hypersonic flow.

The main body of the volume is divided into three parts. The first section, consisting of three papers, is concerned with three-dimensional boundary layers.

I. Three-Dimensional Boundary Layers

1. The Theory of Three-Dimensional Laminar Boundary Layer by Timan.

The solutions of laminar momentum equations are analyzed by considering flow in two components, i.e., components in free stream direction and cross flow components. The method was designed to be useful where large high speed digital computers, which may utilize the more complete partial differential equations of three-dimensional flow, are not available.

The discussion covers comments on other methods, some yielding similar solutions, and contains several photographs showing flow over ellipsoids in water by means of milk ejected at the surface.

2. The Axisymmetric Boundary Layer on a Long Thin Cylinder, by Glauert and Lighthill.

Two solutions of the boundary layer flow along a long thin cylinder are presented. One solution holds near the leading edge of the cylinder, the other holds as the length approaches infinity. Extrapolations for portions in between are presented in graphical and tabular form.

Among the comments were remarks on the applications of the method in the textile industry, i.e., drag of long threads.

3. On the Stabilization of Three-Dimensional Boundary Layers With Application to Flow Due to Rotating Disk, by Gregory, Stuart and Walker.

A beautiful theoretical and experimental study of three-dimensional boundary layer flow. Utilizing the china clay method, a record of the vortices caused by three-dimensional flow in boundary layer of flow over swept wings and rotating disks is pictured. Experimental work is supplemented by theoretical applications of small disturbance theory to general three-dimensional boundary layer equations yielding a three-dimensional stability theory. Application of this theory to cases other than the swept wing and rotating disk are discussed.

Of these, the third paper is of particular interest mainly because of the fine experimental work and its theoretical correlation. The china clay technique yielded fundamentally revealing pictures of the three-dimensional transition to turbulence.

The second section considers the transition from laminar to turbulent flow, effects of viscosity on the flow over swept wings, and a review of recent British work on boundary layer control.

II. Transition, Viscous, Swept Wing Flow, Boundary Layer Control

4. Contributions on the Mechanics of Boundary Layer Transition, by Schubauer and Klebanoff.

A detailed experimental investigation of the mechanism of transition from laminar to turbulent flow generally confirmed the spot concept of Emmons. It was also shown that the flow behind the spots was "calmed" for a period of time during which transition would not occur. In addition to mean flow measurements, oscillograms of the fluctuation in the boundary layer were measured.

5. The Effects of Viscosity on the Type of Flow on Swept Wings, by Kuchemann.

Three types of flow associated with swept wings are considered; the case where the flow remains everywhere attached to the wings, the case where separation of the flow is present, and the three-dimensional wakes behind wings of finite span. The laminar "bubble" and its effects are considered in the second case. Several examples of possible wake patterns are described. In the discussion of the paper, the details of three-dimensional separation were considered and some additional experimental data was presented by Mr. P. Y. Fink of Imperial College.

6. Recent British Work on Methods of Boundary Layer Control, by Pankhurst.

This description of British boundary layer control includes discussion of suction applied to maintain laminar flow, suction through slots on porous surfaces and blowing through slots to delay separation, and methods of boundary layer control, other than suction or blowing. Included in the last category were boundary layer ramps, boundary layer wedges and various types of vortex generators. The extensive discussion on this paper presents later British work on boundary layer control and specific comments on such items as nose suction for high lift and on detailed comment on the economics of laminar boundary layer control as applied to commercial transports.

In this section, the work of Schubauer and Klebanoff concerning the "Calmed" conditions in the transition region of boundary layer flow describes one of the most exciting and fundamentally new discoveries in recent boundary layer study.

The review of British boundary layer control presents a good picture of early British efforts in both laminar and turbulent boundary layer control, but, as Dr. Pankhurst commented, important additions to the state of knowledge were made available after the paper was written.

The final three papers deal with boundary layers in supersonic flow and particularly with shock-boundary-layer interaction.

III. Supersonic Boundary Layer Flow

7. The Profile Drag of Biconvex and Double Wedge Wing Sections at Supersonic Speeds, by Young and Kirby.

A detailed method for computing the profile drag of a flat plate, two biconvex wing sections and two double wedge wing sections is presented. Results of the calculations are presented in graphical and tabular form for a range of Mach numbers, Reynolds numbers and positions of transition. The trends in drag obtained by varying the above parameters are discussed. Some assumptions made in the theory are discussed.

8. The Interaction Between Shock Waves and Boundary Layer and Its Relation to Base Pressure in Supersonic Flow, by Holder and Gadd.

The interaction of a shock and the boundary layer on a flat plate was experimentally examined with particular emphasis on the effects of Reynolds number, Mach number and shock strength. Numerous sketches show schematically the flow patterns in the vicinity of the shock-boundary-layer intersection. The case in which the shock caused separation of the boundary layer was studied in detail. This type of separation was compared with the separation resulting from the flow passing over a step on the surface. Finally the conditions leading to boundary layer separation were examined in relation to base pressures on supersonic bodies.

9. Some Effects of Shock-induced Separation of Turbulent Boundary Layer in Transonic Flow Past Airfoils, by Pearcey.

The effects of the shock-induced separation on two-dimensional airfoils are considered as the free stream velocity is increased through the transonic regime.

The effects on the present disturbance of the shocks as they pass across the surfaces to the trailing edge are described in detail. Practical applications of the study and its extension to finite swept wings is investigated. Numerous data, pressure distributions, downstream pressure recovery factors and Schliern photos are presented.

The last two papers present the basis for a better understanding of the mechanics of shock-boundary layer interaction and would form a good starting point for experimental work concerned with the effects of boundary layer control on shock induced separation.

The volume presents a detailed picture of the present knowledge of the boundary layer, however the presence of a paper on the properties of the turbulent boundary layer and turbulent separation would have greatly broadened the scope of the book. Also the present emphasis on boundary layer control would have justified more attention to this subject. With these exceptions, the volume is certain to be of interest to those concerned with fundamental study of the boundary layer.

JOSEPH J. CORNISH III
Mississippi State College

ATOMIC POWER, edited by Corbin Allardice. 151 pages, plates, diagrams, $5\frac{1}{4} \times 8\frac{1}{4}$ in. New York, London and Paris, Pergamon Press Inc., 1957. $3.50.

An informal panel discussion on atomic power was held at the Eleventh Annual Meeting of the Board of Governors of the World Bank in September of 1956. The remarks of the panel are reproduced in this book.

The interest of the Bank in this subject is given by its president Mr. Eugene Black who notes that over thirty-six per cent of the Bank's development lending has been for electric power projects. Since the world's demands for energy are steadily rising it is necessary that the Bank take into account the possibilities of atomic power as a field for future investment.

The panel was moderated by the editor of this book, Mr. Corbin Allardice, who is the Bank's Advisor on Atomic Energy and was formerly Executive Director of the Joint Committee on Atomic Energy to the U. S. Congress. The panel was composed of four members. The United Kingdom was represented by Sir Edwin Plowden, Chairman of the Atomic Energy Authority and Sir John Cockcroft, Director of the Atomic Energy Research Establishment at Harwell. Representing the United States were Admiral Lewis L. Strauss, Chairman of the Atomic Energy Commission and Mr. W. Kenneth Davis, Director of the Division of Reactor Development of the A. E. C. The remarks of the panel members

are followed by those of Professor Francis Perrin, High Commissioner of the French National Atomic Energy Commission.

The panel members present clear pictures of the United States and British atomic energy programs in some detail. Professor Perrin outlines the French program. It is interesting to note the basic difference between the British program and that of the United States. As pointed out by Sir Edwin Plowden an increase in energy demands of up to forty per cent is anticipated in Britain over the next fifteen years. The easily worked coal has now all been used and future supplies will only be obtained with continually rising costs. Increased use of oil means increased imports and a strain on balance of payments. Therefore the nuclear power plant even in its present or near future stage of development is more attractive economically than in the U. S. where cheap reserves of conventional fuels are still available. The British program calls for a total nuclear capacity of significantly more than two million killowatts by 1965. Admiral Strauss shows that in the United States nuclear power is faced with stiffer competition from conventionally fueled plants than in most areas of the world. The U. S. program thus favors the development of reactor technology rather than the construction of present day plants.

The remarks of the panel are ably summarized by Mr. Allardice. He has also supplied some seventy pages of background material on subjects pertinent to the discussion such as radioisotopes, atomic power, research reactors, etc. Of particular interest is his discussion of the economics of nuclear power.

Appendices are included which state the official American and British policies on nuclear fuel agreements with other countries.

M. P. NORIN
The Franklin Institute Laboratories

PHYSICAL CHEMISTRY, by E. A. Moelwyn-Hughes. 1295 pages, diagrams, 6 × 9¼ in. New York, London and Paris, Pergamon Press Inc., 1957. Price, $15.00.

Professor Moelwyn-Hughes needs no introduction to physical chemists, his "Kinetics of Reactions in Solution" now being accepted as a standard work on the subject. *Physical Chemistry* is not a new edition of a previous work, but an entirely new book, although it did have its beginnings in his early "Introduction to Physical Chemistry," issued by the Cambridge University Press in 1940. The new work is the result of the author's early determination to write a text book of his own, after unsatisfactory experiences with published works on the subject.

The size of the book is impressive, consisting of 1295 pages and twenty-four chapter headings. While the insertion of excellent problems at the ends of the chapters does permit one to call this book a textbook, yet to many readers it will be considered a reference book. In the opinion of the reviewer this will be its chief use.

The subject matter begins with the kinetic-molecular theory, which is treated in the first two chapters. The first chapter considers the experimental foundations, and the second chapter the mathematical formulations of the theory. This method of treatment enables the reader to cover the important facts in the first chapter, and then to plunge into the more mathematical portion in the second chapter. While the author is thorough in his mathematics, he is lucid, and does not hesitate to explain a symbol or term which might not be familiar to the reader.

The third and fourth chapters, on quantum theory, are treated in the same manner, from experimental evidence to mathematical formulation. The readability of these chapters is very good. The reader is surprised to find that he understands points he previously found confusing. This is found when reading such authors as Brönsted. The reviewer ascribes this phenomenon to the fact that the writer really understands his subject.

In chapter five, The Chemical Elements, the reader is introduced to the periodicity of atomic characteristics, and then, gradually, by way of the experimental evidence, to the structure of matter and radioactivity.

Chapter six is a conventional chapter on chemical thermodynamics, the nomenclature following closely that of Lewis and Randall, with slight modifications. This chapter leads naturally into chapter seven, Intermolecular Energy and chapter eight, Partition Functions.

In chapters nine, ten, eleven, and twelve, which form a related group, the properties of molecules are studied by means of their reaction to various types of radiation and to elec-

trons. The chapter headings are informative: Monatomic Molecules, The Dispersion of Light; Diatomic Molecules, The Absorption of Light; Triatomic Molecules, The Raman Effect; and Polyatomic Molecules, The Scattering of X-Rays and Electrons.

The states of matter are now considered. Chapter thirteen, The Crystalline State, chapter fourteen, The Gaseous State, chapter fifteen, The Metallic State, chapter sixteen, The Liquid State, chapter seventeen, The Dissolved State, chapter eighteen, The Ionic State, and chapter nineteen, The Interfacial State, apply a fresh treatment to these parts of physical chemistry which certainly have needed a fresh approach. The subject matter in these chapters is so arranged that they can be read independently, as in any reference work.

Chapters twenty and twenty-one cover chemical equilibria in homogeneous and heterogeneous systems, and chapters twenty-two, twenty-three, and twenty-four, cover the kinetics of reactions in gaseous systems, heterogeneous and photochemical systems, and in solution.

At the end of the book are a number of additional problems. Ten appendices give supplementary mathematical detail, which appears much more graceful here than when inserted in the text, to the distraction of the reader.

This book, which has been printed in Ireland, is not too well bound, and the paper seems to be rather thin for the purpose, there being a definite tendency for print to show through. One could hardly object to the make-up or the printing, however, which are excellent.

Altogether, *Physical Chemistry*, by Moelwyn-Hughes, is an excellent example of the meticulous care with which our British scientist friends do their work. Certainly, the American physical chemist would profit by having this work in his library.

GEORGE S. GARDNER
American Chemical Paint Co.

CHEMISTRY OF THE RARE RADIOELEMENTS, by K. W. Bagnall. 177 pages, diagrams, 5½ × 8½ in. New York, Academic Press Inc., 1957. Price, $5.00.

This book is concerned with the chemistry of the extremely radioactive elements of the Periodic System. These elements are polonium, astatine, francium, radon, radium and actinium. Although weighable amounts of these materials are a rarity in most chemical laboratories, their chemistry has been studied by the various specialized techniques presented in the twelve chapters of this book.

Polonium which has now become available in the pure state through the neutron bombardment of bismuth is covered in the first eight chapters. These include separation and solution chemistry, handling techniques, electrochemistry and a discussion of halides and oxides. These chapters give the reader an insight into the painstaking procedures which are necessary in order to understand the basic chemistry involved. The researcher is often confronted with the problem of using a microchemistry or he may handle larger amounts with the ever present danger of hazardous radiation levels. These chapters on polonium are the most extensively treated in the book and the author has made excellent use of references. This treatment of polonium chemistry is an excellent one and is well worth the large amount of space (approximately one half of the entire text) which has been devoted to it.

Chapter nine is devoted to astatine, francium and radon. Its interesting that the chemistry of francium and astatine have been carried out in trace concentrations (10^{-11}–10^{-15} molar solutions). Some indication of the problem involved in performing chemical operations with these elements can be realized when one recalls that the longest lined isotopes have half lines of 21 minutes (francium) and 8.3 hours (astatine). The author points out that only 69 milligrams of astatine is present in the outermost mite of the earth's crust! Despite the rarity of these elements researchers have been able to learn something of the oxidation states and the anodic deposition potential of astatine.

The experiments dealing with francium chemistry are of necessity few in number; however, co-crystallization studies have been performed and a few precipitation studies have been performed.

Radium and actinium are covered in chapters ten and eleven. The newer methods of separation of radium from barium, ion exchange and electrochromatography and the use of electronic pulse height analyses in deter-

mining radium activity in the presence of daughter elements is presented. Chapter eleven deals mainly with actinium with emphasis on its separation from lanthanium, thorium, cerium and radium.

Dr. Bagnall's work probably contains the most completely consolidated references dealing specifically with the chemistry of these particular heavy radioelements. Extensive bibliographies and two appendices are included which include graphs of the four decay series (thorium, neptunium, uranium-radium, uranium-actinium). There is a glossary which is welcome in a book which speaks in a terminology which may be unfamiliar to many readers.

This work is a comprehensive source-book and is an important contribution to the field of radiochemistry.

J. B. DREW
The Franklin Institute Laboratories

ELECTRON IMPACT PHENOMENA, by F. H. Field and J. L. Franklin. 349 pages, diagrams, 6 × 9 in. New York, Academic Press, Inc., 1957. Price, $8.50.

This volume, subtitled "The Properties of Gaseous Ions," is in reality a thorough review of the effects of the ionization and dissociation of atoms and molecules under low energy (less than 100 ev) electron bombardment. The main emphasis is upon the fundamental chemical physics of the electron impact phenomena. The authors state that their book is intended both for the novice and for the expert. The novice can read this book with profit; all terms and fundamental principles are explained. The expert can profit from the copious compilations of data and from the fact that this book is the first coherent expression of this somewhat restricted field. Thus it may serve as an excellent reference.

Although much of the data comes from mass spectroscopy, mass spectroscopy as such is not reviewed. The main interest is in the ion source, for it is there that electron impact occurs. The authors point out that modern instruments are not designed for the study of electron impact phenomena and thus, in some cases, older devices gave better results. A brief discussion of magnetic deflection spectrometers is given, but theory and construction of such instruments is not covered.

The methods and reliability of obtaining ionization and appearance potentials from ionization efficiency curves are fully discussed. Then follows a theoretical section in which quantum mechanical fundamentals are discussed without, however, going into an extended mathematical treatment. This chapter includes a discussion of the complicated phenomena observed when large organic molecules are involved. In the chapter titled "Energetic Considerations" are found such topics as heat of formation of ions, activation energies, kinetic energies of ions after electron impact, bond strengths, and negative ions. In the chapter on "Mass Spectral Considerations" topics such as mass spectra of diatomic and polyatomic molecules, multiply charged ions, rearrangement, metastable ions, and secondary processes are covered. The section on rearrangement (Ionic dissociation reactions forming products not formed by simple bond fission) should be of extreme interest to the chemist. Finally, there is a chapter devoted to the application of ion energies to the understanding of ionic reactions in organic chemistry.

In addition to the numerous tables scattered through the text, there is a 78-page appendix listing critical potentials for ions. About two thirds of the data here is for organic material. The pure physicist might feel that, throughout the book, too much space has been devoted to organic substances.

As is evident from the above, the book is well documented with references and data. There is a total of 534 references listed near the indices in alphabetical order according to author. There is a separate author index in which reference is made to both the text and to the reference list. The subject index is excellent; it is nicely classified and includes all ionic substances found in the text.

The authors have made a contribution to the scientific literature which is highly readable and intelligently written.

LEONARD MULDAWER
Temple University

MECHANICAL VIBRATIONS, by Bernard Morrill. 262 pages, diagrams, 6 × 9 in. New York, The Ronald Press Co., 1957. Price, $6.50.

Mechanical Vibrations by Prof. Morrill of Swarthmore College is written for use in the

classroom in teaching a first course in mechanical vibrations to the undergraduate and to the graduate student. It should also prove to be a good text for the practicing engineer who wishes to review the subject. All engineers in this category have been exposed to integral calculus which is necessary to study this text. In view of the fact that many of the older professional engineers and some students studying this text may not have studied differential equations a rather full treatment of the solution of a second order linear differential equation is contained in chapter two.

Since the book is composed of eight chapters, it can convey the contents by listing the titles of these. The first chapter deals with the "Dynamics of a Particle," followed by "Mathematics of a System of a Single Degree of Freedom," "Single Degree of Freedom," "System with Several Degrees of Freedom," "Beams and Shafts," "The Lagrange Equations," "Vibration in a Continuous Medium" and the last chapter furnishes brief introduction to modern analytical methods such as electrical-mechanical analogy, mobility method, and analogue computers.

In each of the chapters there are problems for the student to solve. These problems are of a very practical nature that occur in mechanical engineering. This should be of value to those that have occasion to solve such problems at great intervals. Answers to many are supplied at the conclusion of the text. For those who wish to pursue the subject, an ample bibliography is furnished at the conclusion of each chapter.

The author and publisher should be complemented on the presentation of the subject and furnishing of so many well illustrated figures.

A. O. BERGHOLM
The Franklin Institute Laboratories

ROOTS OF SCIENTIFIC THOUGHT: A CULTURAL PERSPECTIVE, edited by P. P. Wiener and Aaron Noland. 677 pages, $6 \times 9\frac{1}{4}$ in. New York, Basic Books, 1957. Price, $8.00.

A short review of this remarkable book can only suggest its conceptual orientation and sample bits of its substantive content. Of the 38 papers included, five are introductory essays written for the volume, and 33 are reprinted from the *Journal Of The History Of Ideas*, where they appeared during the period 1944–57. Every essay concerns itself with at least one problem in the logic and methodology of science, but the theme that imparts organic unity to the collection is best expressed in the Editors' own words. It is "the extralogical components and their affiliations with other cultural developments in the history of the arts, of social institutions, of religion, and of philosophy. By analyzing minutely the complex interrelations of the scientific with the broader culture . . . the historian of ideas aims to shed light on the role played by scientific ways of thinking in shaping the course of civilization."

Of the five papers on "The Classical Heritage," two of broad interest are Heinrich Gomperz's "Problems and Methods of Early Greek Science" and Ludwig Edelstein's "Recent Trends in the Interpretation of Ancient Science." Gomperz analyzes the intellectual climate of the Pre-Socratic scientist-philosophers with regard to the problems they discussed, the methods of explanation they used, and their research methods. Edelstein's review is of Cohen and Drabkin, "A Source Book of Greek Science" (1948), and of Benjamin Farrington's Marxist work, "Greek Science" (1944/1949). This refutation of Farrington's claims is "essential reading" in this present era of ideological warfare.

In his introduction to the essays that recount the progress "From Rationalism to Empiricism," A. C. Crombie shows that the claim of a halt to scientific thought during the Middle Ages is unfounded, and he adduces six Mediaeval contributions to the development of natural science. The nine other essays in this group contain two classics by J. H. Randall, Jr., "The Place of Leonardo da Vinci in the Emergence of Modern Science," and "Scientific Method in the School of Padua." Alexandre Koyre approaches the nature of theoretical constructs via a defense of Galileo's hypothetico-deductive method, which he sees as a development of the Platonic rationalist's search for eternal laws of Nature.

There are three papers by Edgar Zilsel, and a reply to Zilsel by A. C. Keller, that should be read as a conceptual unit. Zilsel (1891–1944) was an extremely able scholar and a Marxist. Even his critics acknowledge his fruitful studies and the challenge of his in-

sights. As a historical materialist and economic determinist, he interpreted the scientific achievements of the Seventeenth Century as the consequences of razing a socio-economic barrier between "lower class" artisans and "respectable scholars" (sic.). Three of his best known papers are here: "The Origins of Gilbert's Scientific Method," "The Genesis of the Concept of Scientific Progress," and "Copernicus and Mechanics." In contrast to most Marxist interpreters, Zilsel had a deep appreciation of democratic ideals and was influenced by the humanitarian views of Karl Mannheim and other "sociologists of knowledge." Keller's paper, and remarks by Randall and Edelstein, supply the reader with corrective critiques.

Among the eleven essays on aspects of the "scientific revolution," there is room to mention only a few: Marjorie Nicholson's outstanding literary-scientific treatment of "Kepler, the *Somnium*, and John Donne"; E. W. Strong's "Newton's Mathematical Way"; and "Bacon's Man of Science", by M. E. Prior, wherein is discussed the moral and cultural responsibility Bacon sought for in his "ideal scientist."

A dozen papers discuss the transformation in scientific thought "from the world machine to cosmic evolution." S. E. Toulmin's paper on Priestley and Lavoisier focuses on the methodological issue of "crucial experiments." There are two papers on biological science, Maurice Mandelbaum's "Scientific Background of Evolutionary Theory in Biology" and A. Ellegård's "Darwin's Theory and Nineteenth-Century Philosophies of Science." Ellegård treats of Whewell's and Mill's positions on evolution, and the role of popular literature and the press during that period.

I. B. Cohen's excellent review of "Some Recent Books on the History of Science" lists references dated as late as 1953.

To my knowledge this is a unique collection, and its scholarly value is obvious. It must be studied, not sampled, for as the Editors point out, "breadth of vision and free play of imagination are as vital as economic factors for scientific growth and cultural survival." This book should be in every high school library as well as on the shelves of scholars.

SAMUEL E. GLUCK
Bonded Scale and Machine Co.

GRAIN BOUNDARIES IN METALS, by D. McLean. 346 pages, diagrams, 5¼ × 8¼ in. New York, Oxford University Press, 1957. Price, $8.00.

Grain boundaries have been the subject of controversy, research, and papers for many years; however, this is the first textbook devoted entirely to this study. The first two chapters dealing with the history and modern theory of grain boundaries serve very well to give the reader a good background on the subject and bring him up-to-date on some of the present day concepts. None of these concepts is as yet entirely satisfactory inasmuch as they cannot explain many of the experimental results.

Chapters are devoted to energy considerations and solute concentration at grain boundaries including the influence of boundaries on deformation, diffusion, and brittleness. The section on intergranular brittleness disposes of low temperature brittleness in only twenty-five pages. There has been probably more written on this one subject in the literature than most of the other subjects combined. Thus, the author's treatment is rather incomplete, especially with respect to the extremely important tough-to-brittle transition in iron.

The migration of grain boundaries, while a major factor in the problem of recrystallization, is given only perfunctory consideration.

The author discusses the problem of subboundaries which separate blocks 10 to 100 microns across and which are slightly misoriented with each other. These boundaries are not grain boundaries but are directly related to the general subject. They have been under intensive study for the past several years and information gained from such study has helped considerably in advancing the general state of knowledge on the nature of grain boundaries themselves.

The book is directed primarily toward metallurgists. On the whole it is well worth their time and should serve as an excellent reference text. The author has done a creditable job in tying together the available literature on grain boundaries. He has organized the subject matter well and has presented it in a clear concise manner. The book has a comprehensive bibliography which appears as

footnotes at the bottom of each page. There are both a subject and an author index.

R. L. SMITH
The Franklin Institute Laboratories

CONTRIBUTIONS TO THE THEORY OF GAMES, VOLUME III, Edited by M. Dresher, A. W. Tucker and P. Wolfe. 435 pages, diagrams, 7 × 10 in. Princeton, Princeton University Press, 1957. Price, $5.00 (paper).

This volume, appearing as Study 39, in the Annals of Mathematical Studies, is a welcome companion to the previous two volumes on contributions to the theory of games [Studies 24 and 28]. Preparation of this study was the joint responsibility of members of the Department of Mathematics of Princeton University and of RAND Corporation. The reviewer feels that the study should be in the library of those that are interested in the mathematical theory of games.

This study investigates certain classes of infinite two-person zero-sum games. It establishes the existence of and presents methods for constructing solutions. The papers are of two types, those in which games are presented in "normalized form" and those in which games are presented in "extensive form." Most of the games studied belong to the class of multi-move games; infinite games which are built up out of a set of "components" which are games or game-like structures, themselves of finite length. If one component of such a set is a "start," and the outcome of each of the components is an instruction to play another component, possibly together with a numerical payoff, then the entire infinite game is recursively defined by the set of rules for the components.

Part I is devoted to games built out of repeated play of simultaneous-move games. It is supposed that each component is already a game in normalized form, each player being uninformed of the other's present choice but completely informed of all choices made in components previously played. The games studied in the five papers of this Part are of the "survival" type where two players begin playing with initial resources and continue until one is ruined.

The games of Part II are again multi-move games, but have perfect information for both players; not only is each player informed of all previous choices, but each component is a game of perfect information. There exist optimal strategies for infinite games with all plays of finite length and perfect information.

The games of Part III carry a weaker information structure than those of Part I: At the time of choice, a player is only partially informed of the component he is playing. The information patterns considered here are those in which a player discovers his opponent's moves after a certain built-in time delay. Unlike games of perfect information, games with a lag of more than one move cannot be decomposed into sub games, and cannot in general be solved by a series of move-by-move optimizations. An interesting "pursuit" game, with an information lag of precisely two moves, is solved, and is a special form of the classical military problem of predicting the position of a maneuvering target.

The papers of Part IV deal with games given directly in normalized form on the unit square. Analytic tools, as well as methods used to solve games in extensive form are employed. A typical game solved is the "m by n bullet silent duel," where two duelists have guns with m and n bullets, respectively.

Part V is devoted to the study of games with a continuum of moves. The previous parts investigate game models that were approximations to models of competitive situations which take place over a time period. Instead of formulating problems in the classical discrete formulation, they are studied in a continuous framework, with the positions of the competitors described at any moment by continuous variables, and their strategic decisions given as functions of continuous variables. The latter are called differential games by various authors. The three papers of this part investigate conditions for existence of values and optimal strategies of certain differential games.

BURTON V. DEAN
Case Institute of Technology

HOW TO MAKE A TELESCOPE, by Jean Texereau, translated by Allen Strickler. 191 pages, illustrations, 6¾ × 10 in. New York, Interscience Publishers, Inc., 1957. Price, $3.50.

Beginners in the art of telescope making feel that there must be some great secret that

allows the expert to finish his mirror easily. This is only a dream. There is no substitute for work and experience except help from an instructor. In local telescope clubs this is easy to obtain. The individual working alone has found the literature hard to find and when obtained difficult to interpret and follow. This has finally been resolved.

Jean Texereau working with the amateurs of Paris wrote his experience into a series of how-to-make articles in *L'Astronomie*. This material has been carefully translated by Allen Strickler and now stands as a great aid to our growing telescope fraternity.

The basic indexed outline along with a fine index replaces the instructor. Written logically, with carefully paralleled illustrations, one trots along the telescope making highway with few red lights.

How a telescope is made, why it works, how accurate the optics should be, comparison of reflectors and refractors, optically and costwise. A few pages cover the subject for beginners and review it for the old experienced hand, and several pages are given on the perfect objective plus a good exposition on the Rayleigh limit. Also discussed are principal types of telescopes, reflectors *versus* refractors as an amateur telescope. The suggestion of the 8-in. instrument as the standard is supported by our own experience. Small mirrors are not necessarily easier to produce an 8-in. one is a man-size job.

Glass and abrasives are covered in complete detail. Convincing reasons should give the novice the support that seems necessary at the beginning of the sometimes fearful work. To know is to have confidence—the basis for comfortable glass working.

The Lyotgraphs give visible evidence that polishing requires careful attention to details and should not be rushed. Optical surfaces are really works of art. The sections on Foucault testing, mirror figuring and proving are exceptionally fine and repay study even by experienced operators.

The pitch lap technic follows Ritchey's method and may be a little slow for small mirrors. The waxing of the pitch surface could well be left out. Too much wax tends to carve the glass surface.

I would suggest that you buy and read this book, before purchasing or making a telescope.

EDWIN F. BAILEY
Amateur Telescope Makers

PUBLICATIONS RECEIVED

The following books have been received in the JOURNAL office for review. Lack of space prevents publication of more than this listing.

ELEMENTARY QUANTITATIVE ANALYSIS, by Harold F. Walton. 342 pages, diagrams, 5½ × 8½ in. New York, Prentice-Hall, Inc., 1958. Price, $5.75.

PHYSICAL METHODS IN PHYSIOLOGY, by W. T. Catton. 375 pages, plates, diagrams, 4¼ × 7¼ in. New York, Philosophical Library, 1957. Price, $10.00.

IN SEARCH OF REALITY, by Viscount Samuel. 229 pages, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $7.50.

GENERAL CHEMISTRY, by L. E. Young and C. W. Porter. Fourth edition, 628 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $6.75.

THE NEUROSES AND THEIR TREATMENT, edited by Edward Podolsky. 555 pages, 5½ × 8 in. New York, Philosophical Library, no date. Price, $10.00.

MOTION AND TIME STUDY, by Ralph M. Barnes. Fourth edition, 665 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $9.25.

DAILY RIVER STAGES, VOLUME LI 1955. 175 pages, 8½ × 10½ in. Washington, D. C., Government Printing Office, no date. Price, $1.00 (paper).

BOOK NOTES

HISTORY OF THE BRITISH IRON AND STEEL INDUSTRY, by H. R. Schubert. 445 pages, plates, diagrams, $6\frac{1}{4} \times 9\frac{3}{4}$ in. London, Routledge & Kegan Paul Ltd., 1957. Price, 60s.

Covering the period 450 B.C. to 1775 A.D., this is the first comprehensive history of the British iron and steel industry up to the introduction of steam power.

Part One covers the first appearance of ironworking in the Bronze Age, by the Celtic people. The author cites as examples articles subsequently brought to light centuries later, and pinpoints the locations of these ironworking communities. Also in Part One are treated the iron industry during the Roman occupation of Britain, during the Anglo-Saxon era (up to 1066) and the destruction of the industry during the Norman period, with the subsequent revival in medieval times. Part One brings the history up to about 1500, the final phase of the direct process.

Part Two is devoted to the indirect process —its introduction, expansion, the development of the blast furnace and the smelting process.

The work is a scholarly treatment, well documented and well illustrated.

BRIDGES AND THEIR BUILDERS, by David B. Steinman and Sara Ruth Watson. Revised edition, 401 pages, illustrations, $5\frac{1}{2} \times 8$ in. New York, Dover Publications, Inc., 1958. Price, $1.95 (paper).

This revised and enlarged edition of a 1941 work includes descriptive material on the new Tacoma Narrows Bridge and the Mackinac Straits Bridge.

Dr. Steinman is well qualified to write of bridges, for he is a leading authority on bridge design. In 1957, he was awarded the Institute's Levy Medal for his outstanding paper in this JOURNAL, on the design of the Mackinac Bridge.

This work begins with natural bridges and continues through primitive bridges, Roman bridges, the Renaissance, 18th Century bridges on through iron bridges, steel bridges, masonry bridges and girder bridges. A final chapter on The Bridgebuilder in Contemporary Civilization covers the various roles the bridge builder is called upon to play—metallurgist, mathematician, foundation expert, steel erector, artist and as a leader of mankind.

PROCESS ENGINEERING IN THE FOOD INDUSTRIES, by R. J. Clarke. 355 pages, illustrations, $5\frac{1}{2} \times 8\frac{1}{2}$ in. New York, Philosophical Library, 1957. Price, $10.00.

Aimed at the people who are responsible for the management of installed plant and improvements to that plant, this new book emphasizes the understanding of principles and the practical knowledge of the tools and methods used in the food industries.

The text material is classified by unit operations and processes; each of these is attacked from three angles—general principles, description of equipment, and illustrative description of plant used in the food industries. The text is British, so the equipment illustrated and described is all British.

A few of the processes and unit operations covered include: liquid flow, heat exchange, liquid filtration, evaporation and distillation, crystallization, adsorption, centrifugal separation, mixing and heat processing methods.

THE GENERAL THEORY OF ELECTRICAL MACHINES, by Bernard Adkins. 236 pages, diagrams, $5\frac{1}{2} \times 8\frac{1}{2}$ in. New York, John Wiley & Sons, Inc., 1957. Price not given.

This text gives a general treatment of electrical machine theory, in an attempt to coordinate the varied methods used in the past. First given as a lecture course to graduate students at Imperial College in London and to design engineers at Rugby, the book reflects the author's twenty years of experience as a designer and consulting engineer. The author gives credit to Gabriel Kron (some of whose papers have been published in this JOURNAL) for the basic idea of a generalized electrical machine, and he states that this present book

may serve as an introduction to Kron's work on matrices and tensors.

The eleven chapters cover the general theory, analysis of machines, the d-c. machine, control systems using d-c. machines, steady-state vector diagrams of a-c. machines, general equations for a-c. machines, a-c. operation of synchronous and induction machines, symmetrical short circuit of an alternator, synchronizing phenomena and sustained oscillation in synchronous machines, approximate methods for generator and system analysis, and the generalized rotating machine.

BETTER REPORT WRITING, by Willis H. Waldo. 231 pages, diagrams, 5 × 7¼ in. New York, Reinhold Publishing Corp., 1957. Price, $4.75.

This small text is written specifically as a quick reference guide to the important details of effective technical report writing. Aiming to simplify the problems of scientific composition, style, tables and use of words, this handbook presents authoritative facts for the report writer. The three appendices present a vast amount of information on punctuation, abbreviations, symbols, etc. Whatever your field of scientific endeavor, this book on report writing will prove invaluable.

TV AND RADIO TUBE TROUBLES, by Sol Heller. 224 pages, diagrams, 5½ × 8½ in. New York, Gernsback Library, Inc., 1958. Price, $2.90 (paper); $4.60 (hard cover).

A complete guide devoted entirely to diagnosing and correcting electron tube troubles. This workbench manuel employs a unique symptom analysis technique for rapid and accurate solutions to tube problems.

SOIL CHEMICAL ANALYSIS, by M. L. Jackson. 498 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $9.00.

In this specialized text, the most frequently used chemical analyses of soil, as well as procedures for plant inorganic constituents are given. The student will find this an excellent research reference of soil chemistry, fertility and genesis.

CURRENT TOPICS

Thermal Antioxidants for Polyethylene.—A number of new additive combinations which are exceptionally effective in inhibiting the oxidation of polyethylene, the most widely used of all plastic materials, have been discovered at Bell Telephone Laboratories. These protectants are up to ten times as effective as presently-used antioxidants.

Certain substituted thioethers, when used in conjunction with carbon black, have been found to gain exceptional antioxidant activity over their use in clear polyethylene. Conventional protectants tend to lose their effectiveness in the presence of carbon black. The substituted thioethers are formed by the condensation of conventional thermal antioxidants, such as aromatic amines or phenols, with sulfur dichloride. Alkyl, aryl and heterocyclic thiols and disulfides without amine or phenolic substituents are also effective antioxidants when used with polyethylene containing carbon black. About 0.1 per cent of the sulfur compound with 3 per cent carbon black is highly effective in reducing both thermal and photo-oxidation. The carbon black, added to cut down photo-oxidation, accounts for the fact that a great deal of the polyethylene in use today is black in color.

The best conventional antioxidants protect carbon black-polyethylene formulations for less than 200 hr. under accelerated test conditions of 140° C. in an atmosphere of oxygen. Furthermore, the oxidation process becomes autocatalytic—that is, it reaches a point at which it "runs away"—under such test conditions. With several of the new protectants, however, oxidation proceeds at a slow, even rate and does not become autocatalytic even after 2000 hr. at 140° C. This is equivalent to much more than 20 years of normal service life.

Some of the new additives, such as polymeric organosulfur compounds including thiols and the polymer from 1, 10-decamethylenedithiol, offer possible other advantages. By control of the molecular weight of the antioxidant, its diffusion rate in polyethylene can be varied. Also, by varying the size of the repeating unit in the antioxidant, the concentration of the active groups can be changed without necessarily affecting their diffusion rates. A high concentration of functional groups in comparatively high molecular weight additives allows maximum protection over longer periods of time. Finally, high molecular weight antioxidants can possibly be added in large enough amounts to act as plasticizers for polyethylene.

Detection of Radioactive Contamination.—Location of radioactive contamination in areas too "hot" for effective use of detection instruments has been made possible by a pin-hole camera fashioned from lead and uranium at the Knolls Atomic Power Laboratory (KAPL), operated by the General Electric Company for the Atomic Energy Commission.

The lead camera also can detect small sources of radioactivity which cannot be sufficiently pin-pointed by detection instruments, John Payne, General Electric engineer who designed the camera, said. In addition, it can be used to make conventional photographs of contaminated objects

which would fog film in an ordinary camera.

The KAPL camera was developed just in time to aid in cleaning up contamination resulting from a nuclear accident that happened in December, 1953, at Canada's Chalk River atomic plant, Payne said.

There, engineers used the lead camera design to build an identical one for use in one of the most difficult jobs in the cleanup, locating "hot" spots on apparatus that had to be dismantled.

Pointed toward the general area of contamination, the camera recorded high-level radioactivity sources on film in minutes rather than days needed to locate these pockets of radioactivity by means of conventional detection instruments.

Payne, supervisor of equipment development at KAPL, explained that basically the camera is of the simple pin-hole variety that any student in high school physics class might build. However, it carries two types of film. One makes conventional light negatives and the other an X-ray film, records atomic radiation.

After development, the X-ray film can be superimposed on the conventional film to pin-point the location of dangerous radiation sources.

Usually about 12 minutes is needed to expose the conventional film and at least an hour for the X-ray film. In many areas of high-level radioactivity where this camera would be used, Geiger counter and other detection instrument operators would not be allowed to enter or might have to spend days locating various radiation sources.

Payne said that the entire camera is approximately the shape and size of an old-fashioned box camera. Outside dimensions of the lead box, he said, are $4\frac{3}{4}$ in. long, 5 in. high and $3\frac{1}{2}$ in. wide.

The lens barrel is made of uranium because uranium is denser than lead and can better limit the entrance of atomic rays to the $\frac{1}{8}$-in. aluminum lens placed inside the barrel. A pin-hole only thirteen and one-half one-thousandths of an inch in diameter in the aluminum lens admits light rays.

Conventional film for recording light rays is placed in the camera nearest the lens. Behind it is a sheet of X-ray film. Separating these two film sheets is a thin lead plate which actually intensifies the nuclear radiation recorded on the X-ray film, Payne said.

Film holders and lead sheet are easily inserted and removed through an opening in the top of the camera. A metal slide can be inserted to cut off light rays after sufficient exposure of the conventional film. This slide does not interfere with the passage of nuclear radiation during the lengthier exposure of the X-ray film.

In most respects, Payne said, the camera could be operated like any ordinary pin-hole camera for taking snap shots except that it isn't very portable. Over-all camera weight, he said, is 29 lb.

Cellophane Wrapper for Meats.—A new cellophane for wrapping fresh meats, superior to current standard cellophanes and lower in price, is being introduced by the Film Division of Olin Mathieson Chemical Corporation called Olin OF-16, the film was developed in the Film Division laboratories with the specific objective of offering a superior wrap for fresh meats. It retains the appearance of fresh meat better and more uniformly; it provides superior protection; and it handles more easily than standard types.

The new cellophane sells for sixty-five cents a pound, four cents less than current standard meat films.

The problem in meat wrapping is to permit oxygen to reach the meat while keeping out moisture. Oxygen is supplied to the meat when the underside of the cellophane becomes moist. Tests show that Olin OF-16 absorbs moisture four times faster than conventional fresh meat cellophane, permitting oxygen to reach the meat more quickly. The outer coating provides protection from moisture in the stores, showcases and household refrigerators.

Another feature of the new cellophane, tests show, is that moisture spreads evenly on the underside near the meat. Conventional films often bead and thus moisture coverage is spotty. This makes for discolored portions on the wrapped meat. Meat wrapped in Olin OF-16 maintains its color uniformly.

The new cellophane is important to supermarkets because easy handling—the non-blocking characteristics of Olin OF-16 makes it easy to slip a sheet from stock—means more meat packaged each hour. Appearance is important because, in self-service, selling, appearance is the greatest stimulus to sales, he added.

As in most self-service selling, the eye-catching qualities of the package are important in making the sale. The ability of OF-16 to maintain the fresh look of meat, and to package the meat at less cost than today, would make it popular in supermarkets and other meat outlets where the customer is encouraged to wait on herself.

Low-Cost Disposal of Radioactive Wastes.—Globs of custard-like gel may be the long-sought answer to safe, low-cost disposal of certain radioactive wastes. General Electric chemists at the Hanford atomic plant are experimenting with sodium silicate, a chemical which, if mixed with a solution containing aluminum, solidifies into a milky white gel. They hope to succeed in using the gel to capture cesium-137 and strontium-90, two radioactive elements which now are stored underground in expensive steel tanks.

Aluminum, the key ingredient in turning the sodium silicate and liquid wastes to gel, is present in liquid refuse from Hanford's chemical separations plants. It becomes part of the waste mixture when protective aluminum jackets are dissolved from around uranium fuel elements following irradiation inside atomic reactors.

If successful, the technique would permit disposal crews to bury globs of gelled wastes in the ground instead of having to use costly steel burial vaults. Introduced wherever possible as a substitute for the tanks, the gel could result in saving nearly one-half the cost of storing wastes underground.

Chemists in G. E.'s Hanford Laboratories also have found that the gel turns to a clay-like substance when exposed to the heat generated by the various radioactive elements. Even higher temperatures melt the clay which upon cooling turns to a material similar to porcelain. Temperatures of more than 1000° C. will create a glass-like material of the original gel, a substance which may be adequate to safely imprison the radioactive elements until they have harmlessly decayed.

Spray Glass.—Marshall's Spray Glass, a brand new product of the manufacturers of Marshall's Photo-Oil Colors, is a lifetime crystal-clear glossy spray that gives the protection of glass without its disadvantages. This special formulated spray protects, preserves, and beautifies all glossy and matte photos, oil and direct colored photos, Polaroid Land prints,

art paintings, graphic arts, murals, etc. Spray Glass hermetically seals out dirt and moisture which prevents ink, dyes, and color pigments from oxidizing, fading, and discoloration. The spray is claimed to dry in seconds and is odorless. Can be obtained at photo, art, and hobby stores.

Monochrome-TV Ultraviolet-Visible Microscope and Absorption Analyzer.—Neutronics Research Company, Waltham, Mass., has developed the first commercially available Monochrome-TV UV-Visible Microscope and Absorption Analyzer.

A UV-Monochromator supplies the preselected UV or Visible wavelength to a 2300° A to 6000° A apochromatized microscope system. The image is picked up by a UV-vidicon camera, and presented on a TV Monitor. Microspectrophotometry is performed by the selection of any one of the horizontal scan lines and display of its density variation (absorption) across the specimen on a line-analyzer oscilloscope. Magnification is about 10,000 times. The latest developments in electronics synchronization, amplification, and control are utilized for precise operation.

Timing and Recording System for Missiles.—Beckman Instruments Co. of Anaheim, Calif., has announced the Model 5710-41, designed to measure and record the travel time of a missile between points on its trajectory. The pulses generated by movements of the missile are sent to two single and two dual channel Time Interval Meters (TIM's) where they are counted and registered as digital data. A 100 KC crystal-controlled oscillator in a Master Time Base furnishes the clock pulses used to measure the time intervals.

A Digital Scanner scans the counters of all the TIM's and sends the information to a Printer. The Scanner contains the scanning switches for all the counters, two stepping switches to provide identification numbers, and reset circuitry for the entire system. Each time the system is reset and a new series of data is sent through the TIM's, the stepping switches will step one decimal number, starting with 01 through 00, so that time intervals of 100 different missiles can be identified.

A standard eight-digit Printer records the digital data and prints a letter with it which will designate the mode of operation. It will operate directly from externally generated pulses, from data already recorded on tape, or in a test function, checking itself against its own time base.

The eight chassis required for the system are housed in one six-foot high cabinet. The system operates on 117 volts $\pm 10\%$, 60 cycle power.

"Icebox" Used in Metallurgical Studies.—An "icebox" that makes giant snowflake crystals is ready for the Brussels World's Fair this summer. It will be one of several American exhibits. The machine, which in five minutes makes giant crystals 2 to 4 in. across from super-cooled water, is a product of the Division of Engineering and Applied Physics at Harvard University. It allows the spectator to watch the process as crystals form. It also provides a new tool for research which may improve the quality of metals for many uses.

The machine stems from an extensive study of solidification; this is part of the research program of the Division of Engineering and Applied Physics of Harvard University. Those working on the topic include Professor Bruce Chalmers, Assistant Professor Richard Davis, Dr. Charles Elbaum and Dr. Kenneth Jackson.

The Harvard device freezes water in a way quite unlike the kitchen refrigerator. In the usual freezing process, water is cooled to the freezing point, and the water remains at 32° F. while it is freezing. The scientific "icebox" supercools water several degrees below the freezing point. Freezing does not begin until the tank is "seeded" by a tube which is cooled to a very low temperature at the appropriate time.

At this point, crystals begin to form in the water, looking, as one scientist put it, like "amphibious snowflakes." They grow very rapidly into long tree-like structures called "dendritic crystals." The crystals are viewed by polarized light which shines through the tank from the back; a second layer of polarizing material between the crystals and the observer causes the growing ice crystals to appear colored. The colors produced run through the whole spectrum, shading into one another like the colors of an oil slick.

The color depends on the thickness of the ice, the inclination of the light and the crystal, and the positions of the two polarizing screens.

The usefulness of the "icebox" as a research tool lies in the similarity of dendritic crystals formed when the supercooled liquid metal in a casting solidifies. Since the process of formation of the crystals in metals largely determines the strength, toughness, hardness and brittleness of the metal, the formation of crystals in supercooled liquids is of great interest to metallurgists. Water was chosen instead of metal for an experimental subject because of its transparency.

The machine is part of the United States contribution to the International Science Exhibit which will be on display at the World's Fair in Brussels for six months beginning this April; it has two tanks which operate simultaneously, cooling, freezing and heating the water. The tanks are adjusted so that the freezing process will be continuously visible in one tank or the other.

New Tube for Nuclear Research.—A powerful new tool for nuclear research—an experimental electron tube that can detect and measure events that happen in less than one ten-billionth of a second—has been disclosed by the Radio Corporation of America.

The experimental tube, designed for use in helping to identify mesons, antiprotons, and other elementary particles that result from the bombardment of atomic nuclei, was described at the Sixth Scintillation Counter Symposium by Dr. George A. Morton, Associate Director of the Physical and Chemical Research Laboratory, RCA Laboratories. Associated with Dr. Morton in its development were R. M. Matheson and M. H. Greenblatt, members of the RCA Laboratories technical staff at the David Sarnoff Research Center, Princeton, New Jersey.

According to Dr. Morton, the new tube, of the type known as a photomutiplier, incorporates new design principles that result in considerably improved performance in comparison to presently available tubes for similar functions. In laboratory measurements, he said, the experimental tube has been shown to possess "time resolution capabilities at least ten times better than can be achieved with present conventional tubes."

Stating that tests have not yet disclosed the performance limits of the new tube, Dr. Morton predicted that when it is used with suitable circuitry, "it will be capable of time resolutions in the range of a few thousandths of a billionth of a second."

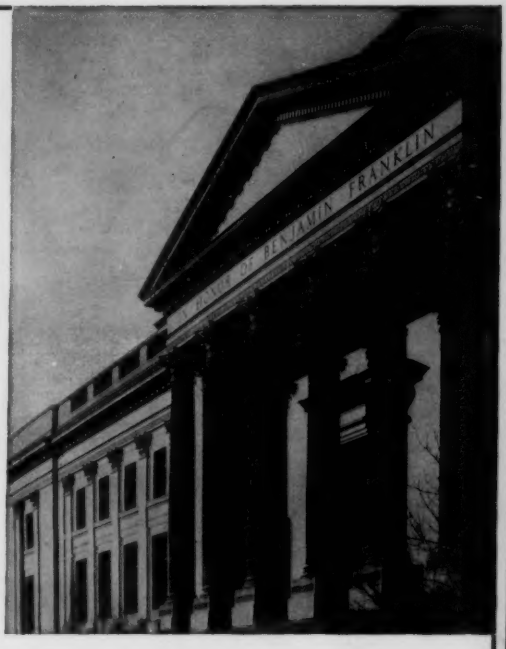
Discussing the use of the new tube,

he pointed out that high-speed photomultipliers are employed as signal amplifiers in scintillation counting, the basic nuclear research technique in which elementary particles are detected by the emission of light as they pass through special counting devices. He explained that one means of identifying the various types of particles is the difference of their velocities, measured as they pass through several scintillation counters separated by precisely measured distances.

Since the particles travel at nearly the speed of light—about 186,000 miles per second—the measurement and comparison of their velocities over a short distance requires tubes that can operate at extremely high speeds.

The experimental tube was developed by Dr. Morton and his associates in a continuing research program at RCA Laboratories under sponsorship of the Atomic Energy Commission. Among previous achievements of the group have been other high-speed phototubes including the RCA 6810 tube used in the discovery of the antiproton at the University of California in 1955.

The Franklin Institute Laboratories for Research and Development



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